Prof. Dr. Michael Günther
Zuzana Bučková, M.Sc.
Dmitry Shcherbakov, M.Sc.
Dipl. Math. Michèle Wandelt, M.Sc.
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Bergische Universität Wuppertal
Fachbereich C – Mathematik und Naturwissenschaften
Angewandte Mathematik / Numerische Analysis

# Numerical Analysis and Simulation I: Ordinary Differential Equations (ODEs)

## Sheet 14 - Repetition 2

**Exercise 31:** *Multi-step schemes*
We consider the scalar initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ $(y : \mathbb{R} \to \mathbb{R})$. Furthermore, an equidistant grid $x_j = jh$, $y_j \doteq y(x_j)$ for $j \in \mathbb{N}$ is used.

**a)** Determine the order of consistency for the BDF2 method

$$\tfrac{3}{2}y_{j+1} - 2y_j + \tfrac{1}{2}y_{j-1} = hf(x_{j+1}, y_{j+1}).$$

Show additionally that the method is stable.

*Reminder*:
The consistency conditions for a general $k$-step scheme

$$\sum_{l=0}^{k} \alpha_l y_{i+l} = h \sum_{l=0}^{k} \beta_l f(x_{i+l}, y_{i+l})$$

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$$\sum_{l=0}^{k} \alpha_l = 0 \quad \text{and} \quad \sum_{l=0}^{k} (\alpha_l l - \beta_l) = 0 \qquad \text{for order 1}$$

$$\sum_{l=1}^{k} \alpha_l l^q = q \sum_{l=1}^{k} \beta_l l^{q-1} \qquad \text{for } q = 2, \ldots, p.$$

**b)** Is the multi-step method given by

$$\tfrac{1}{2}y_{j+1} - 2y_j + \tfrac{3}{2}y_{j-1} = -hf(x_{j-1}, y_{j-1})$$

consistent, stable and/or convergent? What is the connection to part (a)?

**Exercise 32:** *Solutions of Linear DAEs. (part 2)*
Determine the solution of the following linear systems of DAEs (with non-constant matrices).
Investigate which initial values $y(x_0) = y_0$ are feasible.
*Hint*: Observe the two equations of each system. Try to eliminate one unknown ($y_1$ or $y_2$) to obtain a scalar ODE, which is solved analytically.

**a)** $\begin{pmatrix} 0 & 0 \\ 1 & -x \end{pmatrix} \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} 1 & -x \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \sin(x) \\ \cos(x) \end{pmatrix}$

**b)** $\begin{pmatrix} -x & x^2 \\ -1 & x \end{pmatrix} \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

**Exercise 33:** *Space for Additional Questions*