Prof. Dr. Michael Günther
Zuzana Bučková, M.Sc.
Dmitry Shcherbakov, M.Sc.
Dipl. Math. Michèle Wandelt, M.Sc.
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Bergische Universität Wuppertal
Fachbereich C – Mathematik und Naturwissenschaften
Angewandte Mathematik / Numerische Analysis

Numerical Analysis and Simulation I: Ordinary Differential Equations (ODEs)

Sheet 13 - Repetition 1

Exercise 28: *2 stage SDIRK*

Consider the IVP $y' = f(x, y)$, $y(0) = y_0$ and the SDIRK $\begin{array}{c|cc} \gamma & \gamma & 0 \\ 1 & 1-\gamma & \gamma \\ \hline & 1-\gamma & \gamma \end{array}$ with $\gamma = 1 - \frac{1}{\sqrt{2}}$.

a) What are the formulas for the computation of $y_1 \approx y(x_1)$ with stepsize h using intermediate function values k_1, k_2?

b) What are the formulas for Newtons method applied to the nonlinear equation for k_2?

c) Compute the function $R(z)$ for this method and show that the method is A-stable.

Exercise 29: *θ - method*
Consider the IVP $y' = f(x, y)$, $y(0) = y_0$ and the linear multistep scheme
$y_1 - y_0 = h(\theta f(x_1, y_1) + (1-\theta) f(x_0, y_0))$ for $\theta \in [0, 1]$

a) Investigate the stability, order of consistency and order of convergence for this method.

b) Compute the stability function $R(z)$.

Exercise 30: *Runge-Kutta methods*
Consider the IVP $y'(x) = f(x, y(x))$, $x \in [0, b]$, $y(0) = y_0$ with a sufficiently smooth right-hand side $f : [0, b] \times \mathbb{R}^n \to \mathbb{R}^n$ and the following one step scheme:

$$y_1 = y_0 + \frac{h}{4}\Big(f(x_0, y_0) + 3f\big(x_0 + \frac{2}{3}h, y_0 + \frac{2}{3}hf(x_0 + \frac{1}{3}h, y_0 + \frac{1}{3}hf(x_0, y_0))\big)\Big)\ .$$

a) How many evaluations of the right hand side f are needed for one step?
Give the Butcher tableau for the scheme above.

b) Show, that the scheme is consistent with at least order 3.

c) Determine the stability function $R(z), z \in \mathbb{C}$ for the scheme.

d) Is the scheme A-stable or L-stable? Why?
For which kind of problems this scheme might be efficient? (stiff/non-stiff/expensive right hand side/cheap right hand side).