Modelling and Simulating of Rain Derivatives

Master thesis

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Danke

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CHAPTER I

Introduction

Weather influences anyone's life. If it is hot, we turn off the heating and maybe turn on the cooler. If it gets cold we wear warm clothes and turn the heating on again. Maybe a nasty winter even motivates us to do a short holiday on the Maldives.
As weather influences every single person it influences a lot of companies as well. The energy supplier can sell more energy if you turn on your heating. The electricity supplier sells more electricity if you use the cooler. The textile industry profits if you buy a new warm sweater and the hotels in Germany may feel the negative impact of nasty weather in Germany, whereas the Maldivian may feel a positive impact.
But on the other hand the positive impact of a cold winter on the energy supplier can turn into negative if the demand is that high that energy prices raise extraordinarily and the supplier cannot pass the difference to the customer because of price fixing. This phenomenon could appear in other markets as well.

The given examples are obviously not exhaustive. But they give an impression that there is a strong but complex relationship between the performance of a lot of companies and the weather performance. Generally every company itself has to figure out which factors how far influence its profits and gains. That holds for weather specific factors as well. It has to be cleared if the company depends on the weather at all (which is true for a huge amount, see e. g. [BB99]), which weather event or weather combination is decisive, how strong the influence is and how this open weather risk exposure can be hedged. This hedging is advantageous although the company has to pay for passing on a part of its risk. Firstly the income side and/or the cost side becomes more stable. Secondly as ideally the maximum loss or its probability gets lower, the company does not have to provide as much equity as before and can use this "gained" equity for profitable investments etc.

There are already some weather risk hedging tools. A well known idea is the use of weather insurance policies. They are usually individually negotiated contracts between risk seller and insurance. The insurance itself can only sell or decrease their own bought risk by selling it again to another insurance, mostly reinsurance. It is difficult to find another risk which is almost negatively correlated to the risk they have already bought to hedge their risk itself. Therefore the insurance usually demands a high premium if it buys the risk at all. It does not buy risks which ties up too much of the insurance's equity.
The problems with insurances (respectively insurance like protection concepts) lead to the question of more flexible and cheaper hedging tools. In the recent years weather derivatives have become more popular.

Weather derivatives

Derivatives in general are financing tools that derive from an underlying. They cannot exist by itself. Their value is determined by the underlying. A stock option is an example of a stock derivative. The value of the option depends on the price of the stock. Without the underlying stock prices the stock option does not make sense. More details and more exotic examples can be found in [Nel96].
Analogously a weather derivative is a financial instrument which depends on a weather event. Usually this relationship is not direct but between derivative and an index whereas the value of this index depends on the weather.

Weather derivatives are a relatively new concept. The first contracts were made in 1997. The total volume was estimated as 500 million US-$. The first stock exchange for weather derivatives is the Chicago Mercantile Exchange (CME) which established weather indices basing on the weather in 11 different cities in the USA. The big player are energy companies. Slowly insurance companies join. Such different types of derivatives as swaps, options, caps, floor and collars are traded. After a strong growth in the beginning the market for weather derivatives has been growing slowly. Hereby the pricing is considered as one of the main problems (see [BB99]).

As the idea of weather options is quite important in the following, an example is presented.

Example: Important weather indices in the United States are the Heating-Degree-Day-Indices. They measure deviations from a certain reference temperature. We have a closer look on a Heating-Degree-Day-Index (HDD-Index) at location X. It measures if the average temperature is lower than the reference temperature of 65F. The idea behind is that in this case people turn their heating on and their cooling off.
Often such a contract is made for the winter months December, January and February. The HDD-index after these months (if there is no leap year) is

$$HDD^X_{\text{Dez, Jan, Feb}} = \sum_{i=1.12.}^{28.2} (65 - T_i)^+$$

with T_i average daily temperature in Fahrenheit at date i at location X. Assuming that the average temperature at this location is 41 F a put on the HDD with strike $K = 2200$ protects against a warm winter. Its payoff-function looks as follows if one assumes a payoff of 10\$ per index unit

$$f(HDD) = 10 \cdot (K - HDD)^+ \ \$$$

which leads to the following illustration of the payoff versus the index.

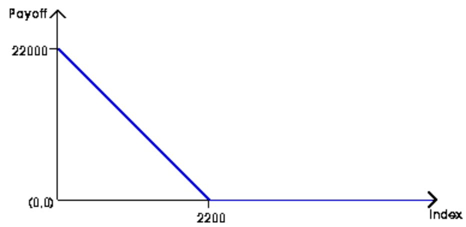

Figure 1.1: Payoff function of a European put

In this example it is not clear how to price the put or more general an appropriate weather derivative. A natural idea is to adapt the pricing tools from stock markets. Here some problems arises.

The probably most popular pricing tool in option pricing is the Black-Scholes-formula introduced in [BS73] and [Mer73]. So a first try is to use the Black-Scholes-concept to price weather derivatives.
The original formula inter alia needs some specific assumptions and conditions (see e. g. [Wil00a] or [Hul00]):

1. underlying is tradable,

2. option is European,
3. option is put or call,
4. there is continuous hedging,
5. underlying is lognormally distributed,
6. risk free interest rate is a constant,
7. no arbitrage opportunities,
8. no transaction costs,
9. no dividends on the underlying.

A situation in which all these conditions hold is called Black-Scholes-world.

If one considers weather derivatives one sees that they do not live in a Black-Scholes world. Admittedly some of the assumptions can be fulfilled. It is quite obvious that one could consider a European style option first. Naturally satisfied is the requirement of no dividend payments as the underlying is a weather event.
Furthermore there are some requirements which do not hold in stock markets either. Therefore one could discuss if they are neglectable. That is e. g. the continuous hedging. In practice only a time discrete hedging is possible and desirable.

These requirements may be manageable. The first of two essential problems is that weather is not tradable. The concept of Black-Scholes is the construction of a riskfree portfolio of the underlying stock and the appropriate option. Then an arbitrage-free argument is applied. In the case of weather derivatives one cannot construct such a riskless portfolio. Thus the conceptual idea of Black-Scholes does not work.
There is also a more general understanding of Black-Scholes formula. One speaks of a Black-Scholes-formula if the underlying is lognormally distributed (refer to [MSS97] for further information). For example there are Black-Scholes-solutions for the Libor market model capletts (see [ABR01] and [AA98]). In this case the construction of a riskless portfolio is not necessary but one gets the price of the option as discounted conditional expectation

$$V(s) = \exp\left(\int_s^T r(u)du\right) E\left(\left(S_T - K\right)|\mathcal{F}_s\right). \tag{1.1}$$

The Feynman-Kac theorem allows to interpret this conditional expectation as a partial differential equation which is the same as we would obtain with a duplication strategy.
Refer to [Øks00] for the relation between expectation of stochastic differential equations and parabolic partial differential equations of second order.
This version of Black-Scholes world is not given in the case of weather derivatives either.

Rain derivatives

Black-Scholes does not work but there have been various attempts to model and price weather derivatives with main focus on temperature, see for example [CW01] and [Sch00].
Considering rain in contrast to temperature additional problems come up. Rain is a local weather event. That means that the fact that it rains at location A does not have to be sign of rain at location B even if they are close. This fact should be born in mind throughout this thesis as it could mean that every location must be modelled individually.
The mathematical formulation is that the correlation between rain at A and rain at B is not (close to) 1. Basing on yearly precipitation the following table shows that this is the case with rain derivatives. Rain is not highly correlated.

	Schleswig	Hamburg	Rostock	Hannover	Düsseldorf	Trier	DAX
Schleswig	1	0.77	0.67	0.54	0.29	0.54	-0.34
Hamburg		1	0.76	0.72	0.33	0.54	-0.32
Rostock			1	0.62	0.18	0.47	-0.12
Hannover				1	0.58	0.55	-0.05
Düsseldorf					1	0.43	0.38
Trier						1	0.08
DAX							1

Table 1.1: Correlation between different weather stations and DAX

A detailed analysis considering the locations city of London and London Heathrow can be found in [Mor00].

Remark: The last column shows the correlation between DAX and yearly accumulated rain at the different weather stations. For calculating the correlation the changes in percent between the statuses at year-end are considered.

The correlations are small and mostly negative. That means that there is diversification potential which makes rain derivatives interesting for portfolio managers.

If one wants to model rain one has to look at rain. Fortunately one does not have to do it by himself. The German weather service (Deutscher Wetterdienst, DWD) has been and is still observing the weather at more than 20 different weather stations in Germany for 35 years and longer depending on the specific weather station. The source for all data presented within this theses is the homepage of the DWD, www.dwd.de. I take special interest in 6 weather stations:

- Schleswig, data available since 1947;
- Hamburg, data available since 1891;
- Rostock, data available since 1947;
- Hannover, data available since 1936;
- Düsseldorf, data available since 1969;
- Trier, data available since 1954.

The percentage of missing data is low. Considering for example Schleswig two records miss. If there is a missing data record it is substituted by random number from a normal distribution around the arithmetic mean of the data at hand.

Topic of this thesis is developing sound models for rain with regard to the situation in Germany and then using these models to price rain derivatives. At his point numerics come into play. Numerical integration and simulation will prove to be necessary to determine the fair value of rain derivatives. The used programming language is Matlab.

The thesis is organised as follows. In chapter 2 a hierarchy of models is set up with regard to an improving mapping of reality. The models are mathematically formulated as stochastic differential equations. Parameter estimators are proposed to calibrate the models to specific locations.
In general the SDEs are not analytically solvable. For that reason numerical integration schemes are presented in chapter 3.
These integrations schemes are used to integrate the SDEs in chapter 4. With means of Monte-Carlo-simulation rain derivatives are priced. Furthermore

the influence of the parameter estimators on the prices is analysed.
Two case studies are shown in chapter 5. The developed models and techniques are applied to realistic examples. Insurance policies and rain options are compared to see if rain derivatives are advantageous in praxis.

CHAPTER II

A hierarchy of models

To get a feeling for rain or more precise precipitation we have a look on some visualisations of rain.

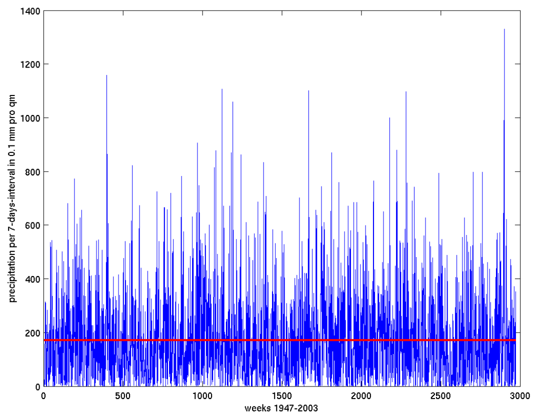

Figure 2.1: Rain, Schleswig, 1947-2003, 7-days-intervals

Figure 2.1 implies that precipitation is strongly fluctuating. There seems to be no kind of regular oscillation but a stochastic noise.

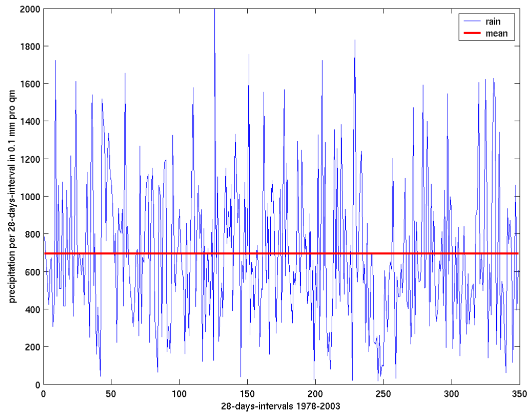

Figure 2.2: Rain, Schleswig, 1980-2003, 28-days-intervals

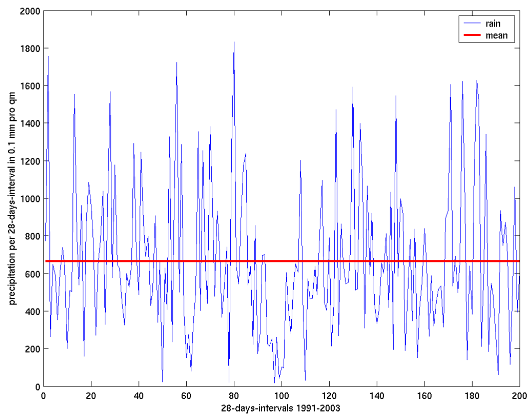

Figure 2.3: Rain, Schleswig, 1992-2003, 28-days-intervals

Considering greater time intervals in figures 2.2 and 2.3 one observes that the process rain fluctuates around a mean. If the process is far away from the mean it seems more likely to move back to the mean than to stride away further.
Thus after having seen these three pictures the first impression of rain is that rain is

- stochastic,

- with high volatility and
- fluctuating around a mean.

Now we have to decide how to describe this phenomenon mathematically: We can choose between discrete and continuous modelling. Within this thesis the assumption of continuity is made. Obviously this assumption is a restriction. There are reasons why this assumption can be considered as maintainable. At first glance rain seems to start suddenly but actually it usually starts slowly and becomes stronger continuously.
Secondly we could measure rain as precipitation in a certain interval, thus we can make it continuous. Figure 2.4 is a visualisation of this idea.

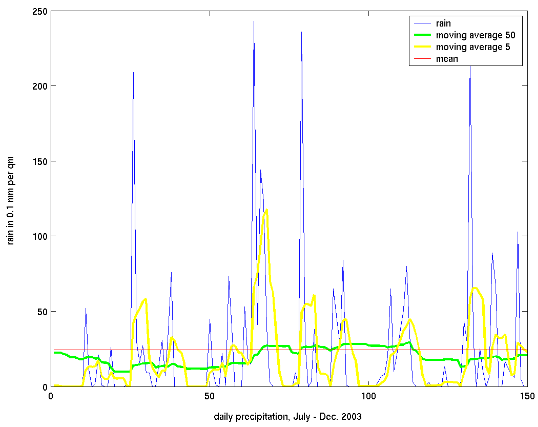

Figure 2.4: Daily precipitation, Schleswig, July - Dec 2003

Thirdly to price rain derivatives eventually we will need numerical integration. Therefore we will have to discretise the continuous model. At this later point we will deal with a discrete model which fits the discrete data.

Figures 2.1, 2.2 and 2.3 gives a first impression about how rain is distributed. Basing on these figures we will proceed as follows. As a starting point we will set up a model which covers these properties of rain. A more detailed analysis of rain will show that this model does not cover all properties of rain. Particularly two problems will arise. Therefore two further models will be set up basing on the first - each of them mapping one more feature of rain. So both of them stand for one step closer to reality. Lastly these two refinements will be combined in a fourth model.

The mathematical means to formulate the models come from stochastic analysis. Especially the stochastic integral is important as the models are phrased as stochastic differential equations.

The models will become usable by developing parameter estimators. Hereby the models can be fitted to particular locations.

2.1 Model 1 - Mean reversion with constant mean

We decided to model rain continously. We observed that rain cannot be predicted which fits every day experience. Furthermore we saw that rain is fluctuating around a mean which means that if rain in a particular period is far away from the long time mean it is more likely to revert back than to depart further.
A mathematical description of the mentioned properties respectively assumptions is given by (2.1)

$$dX_t = \kappa(\theta - X_t)dt + \sigma X_t^p dB_t \tag{2.1}$$

with $t \geq 0$, $\theta > 0, \kappa \geq 0$ and the initial point value $X_0 = x_0$. B_t denotes a standardized Brownian motion. It is the usual formulation of a continuous, stochastic mean-reverting process.
Hereby the stochastic process X_t is the unknown amount of rain at time t. Therefore it makes sense to choose $t \geq 0$ although a generalisation to negative t is possible.
The other parameters θ, κ, σ and p are fixed. The parameter θ is called the mean factor because the stochastic process X_t fluctuates around θ. As there is always a nonnegative amount of rain θ is supposed to be ≥ 0. According to the weather in German we further postulate $\theta > 0$.
κ is the mean-reverting factor. It determines how fast the process X_t returns to the mean. Thus it also makes sense to postulate $\kappa \geq 0$.
The diffusion parameters σ and p describe the volatility of X_t. Having a look on the analytical properties of mean reverting processes one can show that

$$P\left(X_t > 0 : \text{ for all } t\right) = 1 \text{ a. s.}$$

if $p > \frac{1}{2}$ or if $p = \frac{1}{2}$ and $\kappa\theta > \frac{\sigma^2}{2}$. A detailed discussion of analytical properties of diffusion processes can be found in [KT73] and [KT81]. The positivity of mean reverting processes is discussed in [Kah04].

The stochastic differential equation (2.1) describes a stochastic process X_t. An intuitive understanding is that the change in X_t is described by two addends. If we consider only the first addend $\kappa(\theta - X_t)dt$ we have an ordinary deterministic differential equation $dX_t = \kappa \cdot (\theta - X_t)dt$, $X_{t_0} = x_0$ with the solution

$$X_t = \theta + (x_0 - \theta) \cdot e^{-\Delta\kappa} \text{ with } \Delta = t - t_0.$$

If $x_0 = \theta$ then $X_t = \theta$ for all $t \geq T_0$. If $t \to \infty$ the solution tends to θ independently of the initial value. The greater κ the stronger is this effect which fits with the interpretation of κ as factor which determines the speed of mean-reversion.

Secondly there is the addend $\sigma X_t^p dB_t$. That is the stochastic factor. We can think of $\Delta B_{t_i} = B_{t_{i+1}} - B_{t_i}$ as the discretisation of dB_t. It is a normally distributed random variable with mean 0 and standard deviation$\sqrt{t_{i+1} - t_i}$.

An overview about Ito-calculus including a more precise interpretation of (2.1) can be found in the appendix A. Additionally some calculation rules are provided.

2.1.1 Parameter estimation

We have set up a model. The fitting to the given rain data happens by parameter calibration. We want to obtain an unbiased estimator $\hat{\theta}$ for θ and an unbiased estimator $\hat{\kappa}$ for κ. Thus the estimators are supposed to fulfill

$$E\left(\theta - \hat{\theta}\right) = E\left(\kappa - \hat{\kappa}\right) = 0.$$

Ito-Integration of the SDE

$$dX_t = \kappa(\theta - X_t)dt + \sigma X_t^p dB_t$$

leads to

$$X_t = X_0 + \kappa \int_0^t (\theta - X_s)ds + \sigma \int_0^t X_s^p dB_s.$$

We consider the expectation of this equation and get

$$E\left(X_t - X_0\right) = \kappa E\left(\int_0^t (\theta - X_s)ds\right) + \sigma E\left(\int_0^t X_s^p dB_s\right)$$

which simplifies to

$$E\left(X_t - X_0\right) = \kappa E\left(\int_0^t (\theta - X_s) ds\right)$$

because $E\left(\int_0^t f(s, X_s) dB_s\right) = 0$ for any function $f \in \mathcal{V}$ in the Ito-calculus (see [Øks00]). In the following we will use the differential equation version

$$E\left(dX_t\right) = \kappa \cdot E\left((\theta - X_t)dt\right). \tag{2.2}$$

Drift parameters

Estimating θ: A natural estimator for θ is the arithmetic mean

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} X_i. \tag{2.3}$$

Hereby X_i denotes X at t_i. We want to show that (2.3) delivers an unbiased estimator for θ. For that reason we consider the deterministic differential equation given by (2.2)

$$dy(t) = \kappa\left(\theta - y(t)\right) dt \tag{2.4}$$

with some initial value $y(s) = y_s$, $s < t$. Solution of this initial value problem is

$$y(t) = \theta + (y_s - \theta) \cdot e^{-\kappa(t-s)}. \tag{2.5}$$

This enables the following calculation with $\Delta = t_{i+1} - t_i$

$$\begin{aligned}
\frac{1}{n}\sum_{i=1}^{n} X_i &= \frac{1}{n}\sum_{i=0}^{n-1}\theta + (X_i - \theta) \cdot e^{-\kappa\Delta} \\
&= \theta + \frac{1}{n}\sum_{i=1}^{n}(X_0 - \theta) \cdot e^{-\kappa i\Delta} \\
&= \theta + (X_0 - \theta)\,\frac{1}{n}\sum_{i=1}^{n} e^{-\kappa i\Delta} \\
&= \theta + (X_0 - \theta)\,\frac{1}{n}\sum_{i=1}^{n}\underbrace{\left(e^{-\kappa\Delta}\right)^i}_{\in(0,1)}.
\end{aligned}$$

Thus

$$E\left(\theta - \frac{1}{n}\sum_{i=1}^{n} X_i\right) \underset{n\longrightarrow\infty}{\longrightarrow} 0$$

and $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is an unbiased estimator for θ.

Estimating κ: We use (2.2) to show that

$$\hat{\kappa} = \frac{1}{n}\sum_{i=1}^{n} \frac{X_{i+1} - X_i}{(\theta - X_i)\,\Delta} \tag{2.6}$$

is an unbiased estimator for κ if Δ is sufficiently small.

Using (2.5) we can calculate as follows

$$\begin{aligned}
\hat{\kappa} &= \frac{1}{n}\sum_{i=1}^{n} \frac{X_{i+1} - X_i}{(\theta - X_i)\,\Delta} \\
&= \frac{1}{n}\sum_{i=1}^{n} \frac{\theta + (X_i - \theta)\, e^{-\kappa\Delta} - X_i}{(\theta - X_i)\,\Delta} \\
&= \frac{1}{n}\sum_{i=1}^{n} \frac{(X_i - \theta)\cdot\left(e^{-\kappa\Delta} - 1\right)}{(\theta - X_i)\,\Delta} \\
&= \frac{1}{n}\sum_{i=1}^{n} \frac{\left(1 - e^{-\kappa\Delta}\right)}{\Delta} \\
&= \frac{\left(1 - e^{-\kappa\Delta}\right)}{\Delta} \\
&= \frac{1 - (1 - \kappa\Delta) + \mathcal{O}(\Delta^2)}{\Delta} \\
&= \kappa + \mathcal{O}(\Delta).
\end{aligned}$$

Thus $\hat{\kappa}$ is a plausible estimator for small Δ, independently of n.

Unfortunately this estimator turned out to be unsteady. That raises the question why it is unsteady although the estimator is theoretically correct. Problem is the term we neglected by considering the expectation of the SDE (2.1). Actually we have the following expression for κ

$$\kappa = \frac{dX_t}{(\theta - X_t)\,dt} - \frac{\sigma X_t^p dB_t}{(\theta - X_t)\,dt}.$$

The expectation of the second term is zero, $E\left(\frac{\sigma X_t^p dB_t}{(\theta - X_t)dt}\right) = 0$. But what we do in the estimation process is that we consider a discretisation and afterwards take the average of the realisation (time series). If we discretise this second term we obtain

$$\frac{\sigma X_n^p \Delta B}{(\theta - X_n)\,\Delta}. \tag{2.7}$$

In the case if $\theta - X_n$ is close to zero, the expression becomes very large. So the cases with $\theta - X_n$ very small dominate the estimator. To avoid this effect, a modification makes sense:

$$\hat{\kappa_b} = \frac{1}{\# I_b} \sum_{i \in I_b} \frac{X_{i+1} - X_i}{(\theta - X_i)\,\Delta}.$$

with $I_b = \{i = 1, ..., n : |\theta - X_i| > b\}$ and an appropriate choice for b.
Tests show that this estimator is stable, also with respect to the boundary b. Figure 2.5 visualises this effect:

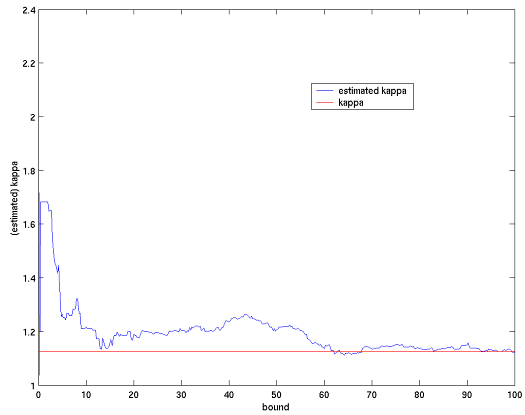

Figure 2.5: κ as a function of the boundary

In this picture the estimation of κ is plotted against the bound on the x-axis. The estimation bases on one simulated path, using the data from Schleswig. One observes that the estimation with a low boundary does not lead to estimators near the real κ, but with increasing boundary the estimation comes close to the real input κ (see also chapter 4).

Thus at this point we can estimate the parameters of the drift term independently of the diffusion parameters.

Diffusion parameters

To estimate the diffusion parameters a small trick is applied (see [Wil00a]). Squaring the stochastic differential equation (2.1) leads to

$$(dX_t)^2 = \kappa^2 \cdot (\theta - X_t)^2 \cdot dt \cdot dt + \kappa \cdot (\theta - X_t) \cdot \sigma \cdot X_t^p \cdot dt \cdot dB_t + \sigma^2 \cdot X_t^{2p} dB_t^2.$$

Applying the calculation rules (A.5) simplifies the squared SDE. One obtains an equation independent of the drift parameters

$$(dX_t)^2 = \sigma^2 X_t^{2p} dt. \tag{2.8}$$

Taking the logarithm of (2.8) leads to

$$\ln (dX_t)^2 \quad = 2\ln(\sigma) + \ln(dt) + 2p\ln(X_t) \quad = a + b\ln(X_t) \tag{2.9}$$

with $a = 2\ln(\sigma) + \ln(dt)$ and $b = 2p$. One observes that there is a linear relation between $\ln(dX_t)^2$ and $\ln(X_t)$. Having the data points from our time series, a and b can be estimated by finding the best fit straight line. Directly σ and p follow from a and b.

2.2 Model 2 - Mean reversion with deterministic mean

In model 1 two assumptions are made: First θ is constant and second rain is a stochastic processes fluctuating around this constant mean. Having a closer look at rain it seems that there are seasonality effects. Figure 2.6 shows the average monthly rain at the weather stations Hannover and Schleswig.

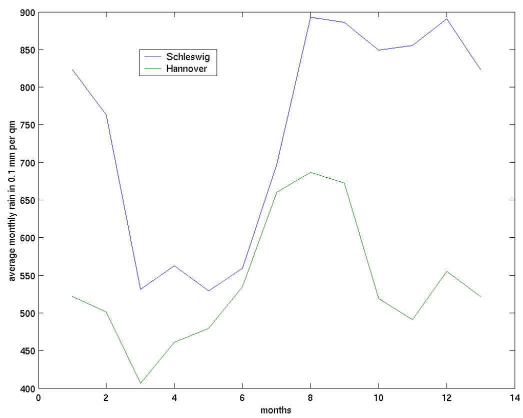

Figure 2.6: Monthly average rain, data series: 1954-2001

One observes that the averages differ. Characteristics like that are not covered in a constant mean. Thus an improvement of model 1 is substituting the constant mean θ by a function $\theta(t)$. That is what is done in model 2.

As an example the time unit month is considered. The rain unit is 0.1mm per m^2.
The SDE

$$dX_t = d\theta(t) + \kappa(\theta(t) - X_t)dt + \sigma X_t^p dB_t \tag{2.10}$$

with $t \geq 0, \kappa \geq 0$ and the initial point value $X_0 = x_0$ describes a mean-reverting-process with a deterministic mean function $\theta(t)$. As in model 1 we postulate $\theta(t) > 0$. See [DQ00] for explanation of the term $d\theta(t)$.

What is a sensible choice for the function θ? As the function is supposed to fit with the data, one has to look at the data again. Figure 2.6 implies that the mean is more like a sine than a constant. So we choose $\theta(t)$ as a sine:

$$\theta(t) = m + \alpha \sin\left(\frac{2\pi(t-v)}{12}\right).$$

Hereby v is the shift on the x-axis (to scale up to months we divide by 12) α determines the oscillation and m is the mean of the sine curve.
Plotting the sine against the real data we see that it is far better than the constant.

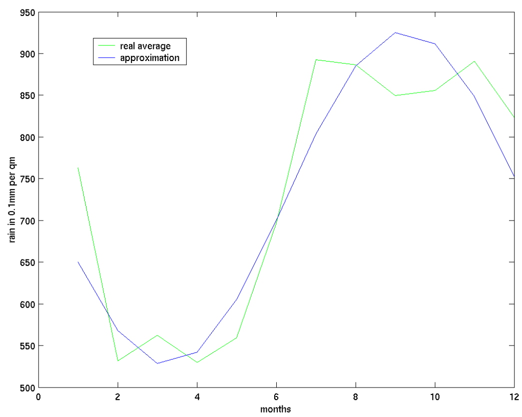

Figure 2.7: Monthly average vs $\theta(t)$, Schleswig, data series: 1954-2001
$\theta(t) = 745 + 200\sin\left(\frac{\pi(t-6.25)}{6}\right)$

But there are some characteristics which are not represented by this sine function. The two notches are striking and not yet covered. Possible adaption is adding a second sine term or more generally to apply a Fourier analysis on the given data:

$$\theta(t) = m + \sum_{i=0}^{n} \alpha_i \sin\left(\frac{(2i+1)\,\pi\,(t-v)}{6}\right).$$

Having a look on the plot of the average values from Schleswig in figure 2.8 one sees that this new θ-function is rather close to the real averages.

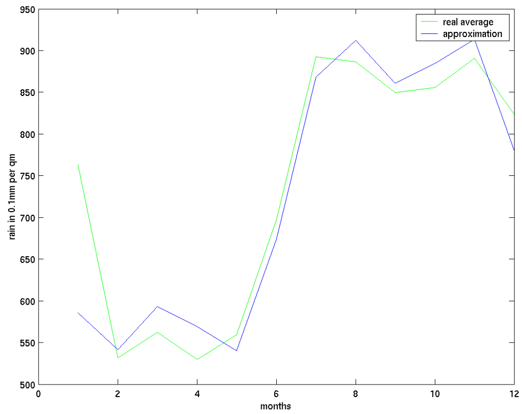

Figure 2.8: Monthly average vs $\theta(t)$, Schleswig, data series: 1954-2001
$\theta(t) = 745 + 200\sin\left(\frac{\pi(t-6.25)}{6}\right) + 70\sin\left(3\frac{\pi(t-6.25)}{6}\right)$

This leads to the question how many sine-terms are necessary. The answer depends on the number and on the length of the time intervals. This topic is dealt with in chapter 4.

Parameter estimation

We want to adapt the methods used for parameter estimation in model 1.

θ-function

We can estimate the parameters of the θ-function, m, α_i, v with least-square method. Therefore the function

$$f(t) = m + \sum_{i=0}^{n} \alpha_i \sin\left(\frac{(2i+1)\,\pi\,(t-v)}{6}\right)$$

is compared with the real averages $D(t)$, $D(t)$ average over all years at t.

E. g. let $i = 1951, ..., 2000$ be the years within the time series. Then

$$D(k) = \frac{1}{50}\sum_{i=1}^{50} R(k,i)$$

holds with $R(k,i) = \{\text{rain in month } k \text{ in year } i\}, k = 1, ..., 12$.

For this thesis the Gauss-Newton method has been implemented to find the minimising parameter constellation in a least-square sense, thus to solve

$$\min_{m,\alpha_i,v} \|f(\cdot, m, \alpha_0, ..., \alpha_n, v) - D(\cdot)\|_2^2.$$

The Gauss-Newton method is for example presented in [Sch97b].

Remark: This thesis deals with 6 weather stations. In every single case the θ-function as chosen above proved to be suitable. The differences between the stations reflect in the different estimated parameters.

Drift parameters

Estimating θ has been already done. But there is still κ to calibrate. Analogously to model 1 the expectation of (2.10)

$$E\left(dX_t\right) = E\left(d\theta(t)\right) + \kappa \cdot E\left((\theta - X_t)dt\right)$$

is discretised and averaged. That leads to the following estimator for κ

$$\hat{\kappa} = \sum_{i=0}^{n} \frac{X_{i+1} - X_i - \theta(i+1) + \theta(i)}{\left(\theta(i) - X_i\right)\Delta}.$$

Here again the modification to

$$\hat{\kappa_b} = \frac{1}{\#I_b}\sum_{i\in I_b} \frac{X_{i+1} - X_i - \theta(i+1) + \theta(i)}{\left(\theta(i) - X_i\right)\Delta}$$

with $I_b = \{i = 1, ..., n : |\theta(i) - X_i| > b\}$ and an appropriate choice for b turned out to be an improvement.

Diffusion parameters

The idea is to use the same trick as above, squaring (2.10) leads to

$$(dX_t)^2 = \kappa^2 \cdot (\theta - X_t)^2 \cdot dt \cdot dt + \kappa \cdot (\theta - X_t) \cdot \sigma \cdot X_t^p \cdot dt \cdot dB_t.$$

According to the Ito-calculation-rules (A.5) one obtains the following relation between $(dX_t)^2$ and X_t:

$$\ln (dX_t)^2 \quad = \quad 2\ln(\sigma) + \ln(dt) + 2p\ln(X_t) = a + b\ln(X_t) \,.$$

That is exactly equation (2.9). Thus the estimation of p and σ works as above.

2.3 Model 3 - Model 1 driven by fBM

Model 1 and 2 implicitly share one characteristic. dB_{t_i} and $dB_{t_{i+1}}$ are uncorrelated. In other words the fact that it is raining today does not affect the probability that it is going to rain tomorrow. Subjectively one would say that this does not fit with reality or mathematically that dB_{t_i} and $dB_{t_{i+1}}$ are correlated. It seems plausible that there are periods when it rains more than usual and that there are periods when it rains less than average. Figure approves that impression.

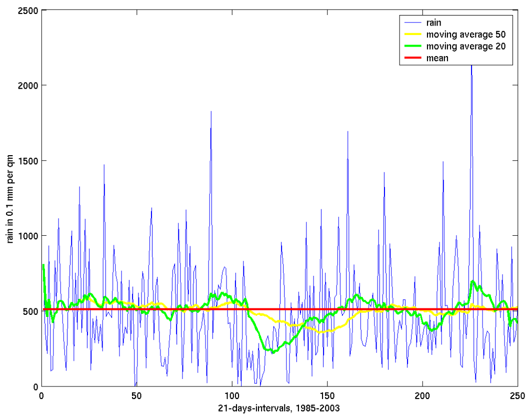

Figure 2.9: Moving average, Schleswig, 1947-2003

Although the moving averages do not strongly depart from the mean there are periods when the medium term average is above or below the longterm mean.

It seems to be worth a try to model a long term relationship. Hereby the model construction allows an ex post test (by parameter estimation) if there is a long term relationship or not.

Model 3 is a refinement of model 1. It does only take into account a constant mean

$$dX_t = \kappa(\theta - X_t)dt + \sigma X_t^p dB_t^H \tag{2.11}$$

with $t \geq 0$, $\theta > 0, \kappa \geq 0, H \in [\frac{1}{2}, 1)$ and the initial point value $X_0 = x_0$.
The long term relationship is determined by dB_t^H. Hereby B^H denotes fractional Brownian motion with Hurst parameter H. One can think of $\Delta B_{t_i}^H$ as the increment of fractional Brownian motion respectively the discretisation of dB_t^H. The Hurst parameter determines the longterm relationship. It originates from analysing the water amount of the Nil river by Hurst. It was discovered that water amount distributions of many rivers are described by Hurst factors $H \in [\frac{1}{2}, 1)$.
Mandelbrot in [Man83] gives an interesting introduction.

2.3.1 Fractional Brownian motion

This subsection particularly benefitted from [MvN68], [Die02] and [DHPD00]. As it is only a very short overview given the interested reader is referred to the sources.

In the originating work [MvN68] Mandelbrot and van Ness defined fractional Brownian motion by its stochastic representation
(2.12)

$$B_t^H = \frac{1}{\Gamma\left(H + \frac{1}{2}\right)} \left(\int_{-\infty}^{0} [(t-s)^{H-\frac{1}{2}} - (-s)^{H-\frac{1}{2}}] dB_s + \int_{0}^{t} (t-s)^{H-\frac{1}{2}} \, dB_s \right)$$

with the Gamma function

$$\Gamma(x) = \int_0^\infty u^{x-1} e^{-u} du$$

and the Hurst parameter $H \in (0, 1)$. B_s again denotes the ordinary Brownian motion which one recovers by setting $H = \frac{1}{2}$. One can compute the variance of B_t^H as

$$Var\left(B_t^H\right) = \alpha_H t^{2H}$$

for a constant α_H. A fractional Brownian motion is called standardised if $\alpha_H = 1$. In the following we will implicitly assume that we deal with standardised fractional Brownian motion which is uniquely determined by

- B_t^H possess stationary increments,
- $B_0^H = 0$, $E(B_t^H) = 0$ for $t \geq 0$,
- $E\left((B_t^H)^2\right) = t^{2H}$ for $t \geq 0$,
- B_t^H is normally distributed for $t > 0$.

Furthermore it is supposed to have continuous trajectories. The Kolmogoroff criterion guarantees that such a version exists.

Short calculation shows that the covariance kernel is as follows

$$\rho(s,t) = E\left(B_s^H B_t^H\right) = \frac{1}{2}\left(t^{2H} + s^{2H} - (t-s)^{2H}\right). \tag{2.13}$$

Note that mean and covariance structure uniquely characterise the finite-dimensional distributions in case of Gaussian processes (see for example [Die02]).

There are two properties of fractional Brownian motion which are particularly interesting to us. That is self similarity and long range dependence. A process X_t exhibits long range dependence if

$$\sum_{n\geq 1} r(k) = \infty$$

holds for $r(k) = cov(X_n, X_{n+k})$. Secondly we call a process X_t self similar with Hurst parameter $H \in (0,1)$ if $\left(X_{at}^H\right)_{t\geq 0}$ and $\left(a^H X_t^H\right)_{t\geq 0}$ possess the same probability law.

As in the case of Gaussian processes the finite dimensional distributions are uniquely determined by mean and covariance structure we can conclude from (2.13) that fractional Brownian motion is self similar.

Furthermore in the case of $H > \frac{1}{2}$ it is long range dependant, too. For explanation see e. g. [Die02].

The increment process $X = \{X_i : i = 0, 1, ...\}$ of fractional Brownian motion is called fractional Gaussian noise and defined by

$$X_i = B_{i+1}^H - B_i^H.$$

The X_i are normally distributed with $E(X_k) = 0$ and $Var(X_k) = 1$.The auto-variance kernel is given by

$$r(t_i) = \frac{1}{2}\left(|i-1|^{2H} - 2|i|^{2H} + |i+1|^{2H}\right). \tag{2.14}$$

Using (2.14) one can show that the covariances are positive if $H > \frac{1}{2}$ and negative if $H < \frac{1}{2}$. If $H = \frac{1}{2}$ the covariances are 0 which corresponds to the independence of the increments of an ordinary Brownian motion.

Furthermore the (fractional) increment process is self-similar. The proof uses the self-similarity of the underlying fractional Brownian motion. As we deal with a normal distributed process it is only necessary to confirm that mean and covariance function of X^H_{mt} and $m^H X^H_t$ are equal. Obviously both expectation values are zero. Therefore only one calculation is necessary:

$$\begin{aligned}
& cov\left(X_{km} + ... + X_{(k+1)m-1}, X_{\ell m} + ... + X_{(\ell+1)m-1}\right) \\
= \; & cov\left(B^H_{(k+1)m} - B^H_{km}, B_{(\ell+1)m} - B^H_{\ell m}\right) \\
= \; & cov\left(m^H B^H_{k+1} - m^H B^H_k, m^H B^H_{\ell+1} - m^H B^H_\ell\right) \\
= \; & cov\left(m^H X_k, m^H X_\ell\right).
\end{aligned}$$

Again we have to define what

$$\int_0^t f(s, X_s) dB^H_s$$

is. A careful introduction to fractional integration would go beyond the scope of this thesis. The concept used is the stochastic integration type introduced by Duncan, Hu, Pasik-Duncan (see [DHPD00]). Essentially fractional Brownian motion is no semi-martingale. Therefore the Ito-formula as it is presented in chapter 2 cannot be applied to stochastic processes driven by a fractional Brownian motion.

2.3.2 Parameter estimation

Again we try to use the methods we have already developed.

Drift parameters

To estimate the drift parameters we again take the expectation of the integral presentation of the SDE (2.11). That is

$$E\left(X_t - X_0\right) = \kappa E\left(\int_0^t (\theta - X_s)ds\right) + \sigma E\left(\int_0^t X_s^p dB_s^H\right)$$

which again simplifies to

$$E\left(X_t - X_0\right) = \kappa E\left(\int_0^t (\theta - X_s)ds\right).$$

There is the same situation as in the case of normal Brownian motion. The estimation of θ and κ can be done as above (independently of the diffusion parameters).

Diffusion parameters including Hurst factor

As well in the case of the diffusion parameters the method used for model 1 and 2 is adapted. Aim is to estimate the Hurst factor H together with the parameters σ and p in one single step. We square SDE (2.11) and obtain:

$$(dX_t)^2 = \kappa^2 \cdot (\theta - X_t)^2 \cdot dt \cdot dt + \kappa \cdot (\theta - X_t) \cdot \sigma \cdot X_t^p \cdot dt \cdot dB_t^H + \sigma^2 \cdot X_t^{2p} dB_t^{H^2}.$$

Using the calculation rules that expression can be simplified to

$$E\left((dX_t)^2\right) = \sigma^2 \cdot X_t^{2p} E\left(\left(dB_t^H\right)^2\right).$$

What is $E\left(\left(dB_t^H\right)^2\right)$? One can calculate

$$\begin{aligned} E\left(dB_t^{H^2}\right) &= Var\left(dB_t^H\right) + E\left(dB_t^H\right)^2 \\ &= Var\left(dB_t^H\right) + 0 \\ &= dt^{2H}. \end{aligned}$$

Thus

$$E\left((dX_t)^2\right) = \sigma^2 \cdot X_t^{2p} \cdot dt^{2H} \tag{2.15}$$

holds and the logarithmic version looks as follows

$$\ln\left((dX_t)^2\right) \quad = 2\ln(\sigma) + 2H\ln(dt) + 2p\ln(X_t) = a(H) + b\ln(X_t).$$

We have got an expression in the three variables σ, p and H independently of θ, κ. We distinguish two cases:

1. **discretisation $\Delta \neq 1$ for dt:**
   We get the three estimators as the minimising triple of

   $$\min_{a,b,H} \| \log\left((X_{n+1} - X_n)^2\right) - a - b\log(X_n)\|_2^2. \tag{2.16}$$

   Again the minimum is computed with the Gauss-Newton algorithm.

2. **discretisation $\Delta = 1$ for dt:**
   In this case the equation is independent of the Hurst factor H

   $$E\left((dX_t)^2\right) = \sigma^2 \cdot X_t^{2p}.$$

   Therefore we can calibrate the parameters θ, κ, σ and p as in model 1 and 2 independently of H. So in this case we can do the model refinement ex post by determining the Hurst factor.

If $\Delta = 1$ one needs a method to calibrate the Hurst factor H. There are many methods to estimate H if fractional Gaussian noise is given. In the existent case the recorded data are not the fractional Gaussian noise itself. The data are assumed to follow the SDE (2.11) driven by Gaussian noise, that is dB_t^H.
At this point we know or at least we can estimate all variables in (2.11) except the Hurst factor which determines the Gaussian noise.

To separate the dB_t^H-term we discretise the SDE (2.11). Actually every discretisation scheme can be used. Within this thesis the explicit Euler is applied. That means for (2.11) with $Y = \Delta B_n$

$$X_{n+1} - X_n = \kappa(\theta - X_n)\Delta + \sigma X_n^p \cdot \sqrt{\Delta} Y_n.$$

We get an approximated sample of dB_t^H by determine the zero point process Y. The Euler scheme leads to a linear equation which can be solved directly. By doing so we obtain from each data series one sample of Gaussian noise. One possibility to estimate the Hurst factor $\frac{1}{2} < H < 1$ from such a sample is the

Aggregated variance method: The method is presented as it is described in [Die02] where the interested reader can find a detailed discussion. The aggregated variance method is applied on a Gaussian noise sample Y_k on $\{0, \frac{1}{N}, ..., \frac{N-1}{N}\}$. The method uses the self-similarity of the sample. It is because of the self-similarity property that the aggregated process $X^{(m)} = \left(X_k^{(m)}\right)_{k\in\mathbb{N}_0}$ with

$$X_k^{(m)} = \frac{1}{m}\left(X_{km} + ... + X_{(k+1)m-1}\right)$$

and $m^{H-1}X$ has asymptotically the same finite dimensional distributions. Especially

$$Var\left(X_k^{(m)}\right) = m^{2(H-1)}Var\left(X_k\right)$$

holds. A sensible estimator for $Var\left(X_k^{(m)}\right) = Var\left(X_0^{(m)}\right)$ for every k is

$$\widehat{Var\left(X_0^{(m)}\right)} = \frac{1}{M}\sum_{i=0}^{[\frac{N}{m}]-1}\left(X_i^{(m)} - \overline{X^{(m)}}\right)^2$$

with

$$\overline{X^{(m)}} = \frac{1}{[\frac{N}{m}]}\sum_{i=0}^{[\frac{N}{m}]-1} X_i^{(m)}.$$

Plotting $\log\left(\widehat{V\left(X_0^{(m)}\right)}\right)$ versus $\log(m)$ one gets an estimator for $2(H-1)$ from the slope of the best fit straight line.

Remark: The given procedure to estimate the parameters of the SDE can be applied to any SDE with the structure

$$dX_t = g(t, X_t)dt + f(t, X_t)dB_t^H.$$

2.4 Model 4 - Model 2 driven by fBM

The next model is the natural continuation from what we have done so far. It combines refinements of model 2 and model 3.

$$dX_t = d\theta(t) + \kappa(\theta(t) - X_t)dt + \sigma X_t^p dB_t^H \tag{2.17}$$

with $t \geq 0$, $\theta > 0, \kappa \geq 0, H \in [\frac{1}{2}, 1)$ and initial point value $X_0 = x_0$. It describes a mean-reverting process with a long-term-relationship and a non-constant mean $\theta(t)$.

Parameter estimation

The parameter estimation happens as in the models above.

Drift parameters

Because of the properties of the chosen fractional integral version

$$E\left(dX_t\right) = E\left(d\theta(t)\right) + \kappa E\left(\left(\theta - X_t\right) dt\right)$$

holds. One obtains the estimator for the drift parameters as in model 2.

Diffusion parameters including Hurst factor

Again squaring (2.17) leads to

$$E\left(\left(dX_t\right)^2\right) = \sigma^2 \cdot X_t^{2p} \cdot dt^{2H}. \tag{2.18}$$

and σ, p and H can be calibrated as in model 3.

Summary

This thesis is a first approach to fit the amount of rain to stochastic differential equations particularly mean-reverting processes. The initial problem is to fit the parameters to the chosen model which is done in this chapter. As we do not know the density of the mean-reverting processes in general we cannot apply a likelihood approach. Therefore we have to do a moment matching. In this thesis we present some ideas to estimate the drift and diffusion parameters as well as we develop techniques to fit the data to an fBM-approach.

CHAPTER III

Simulating the models

If the rain process is given by (2.1), (2.10), (2.11) or (2.17) the price P at time t of a rain derivative is calculated as

$$P(t) = \exp\left(\int_t^T r(u)du\right) E[f\left(X_{t_1}, ..., X_{t_n}\right)] \tag{3.1}$$

with maturity T and riskfree interest rate $r \in \mathcal{C}^1\left(\mathbb{R}^+, \mathbb{R}^+\right)$. This pricing method is known as expectation principle. As none of the SDEs is analytically solvable numerical integration is necessary to compute the price of a rain derivative.

There are various possible stochastic integration schemes. Three different schemes are used within this thesis. The schemes and short derivations are presented in this chapter. Hereby distinction between integration after Brownian motion and fractional Brownian motion is requisite.

3.1 Integration schemes for BM

Most important tool to construct numerical integration schemes is stochastic Taylor expansion. But as only first and second term are necessary for the integration schemes used in this thesis the whole stochastic Taylor expansion has not to be built up. A light version suffices.
An extensive book about this topic is [KP92].

We construct the beginning of the stochastic Taylor expansion by means of Ito's formula. Note that this construction only holds for semi-martingales W, which is a property Brownian motion fulfills, but fractional Brownian motion does not.

Let

$$X_t = X_{t_0} + \int_{t_0}^{t} a(s, X_s)ds + \int_{t_0}^{t} b(s, X_s)dW_s \tag{3.2}$$

be an Ito-process and $f(t, x) \in \mathcal{C}^{1,2}([0, \infty) \times \mathbb{R})$[1]. Then $f(t, X_t)$ is again an Ito-process and it holds

$$\begin{aligned} f(t, X_t) &= f(t_0, X_{t_0}) + \int_{t_0}^{t} \frac{\partial f(s, X_s)}{\partial t} ds + \int_{t_0}^{t} a(s, X_s) \frac{\partial f(s, X_s)}{\partial x} ds \\ &\quad + \frac{1}{2} \int_{t_0}^{t} b^2(s, X_s) \frac{\partial^2 f(s, X_s)}{\partial x^2} ds + \int_{t_0}^{t} b(s, X_s) \frac{\partial f(s, X_s)}{\partial x} dW_s. \end{aligned}$$

Next $a(s, X_s)$ and $b(s, X_s)$ are substituted in (3.2) by its Ito formula representation. That means for the first integral in (3.2):

$$\begin{aligned} &\int_{t_0}^{t} a(X_s, s)ds \\ =\ & \int_{t_0}^{t} \Big(a(t_0, X_{t_0}) + \int_{t_0}^{s} a(u, X_u) \frac{\partial a(u, X_u)}{\partial t} du + \int_{t_0}^{s} a(u, X_u) \frac{\partial a(u, X_u)}{\partial x} du \\ & + \frac{1}{2} \int_{t_0}^{s} b^2(u, X_u) \frac{\partial^2 a(u, X_u)}{\partial x^2} du + \int_{t_0}^{t} b(u, X_u) \frac{\partial a(u, X_u)}{\partial x} dW_u \Big) ds \\ =\ & a(t_0, X_{t_0}) \int_{t_0}^{t} ds + \mathcal{R}_a(t) \end{aligned}$$

with

$$\mathcal{R}_a(t) = \int_{t_0}^{t} \int_{t_0}^{s} a(u, X_u) \frac{\partial a(u, X_u)}{\partial t} duds + \int_{t_0}^{t} \int_{t_0}^{s} a(u, X_u) \frac{\partial a(u, X_u)}{\partial x} duds$$

[1] The idea can easily be extended to functions $f : \mathbb{R} \longrightarrow \mathbb{R}^n$.

$$+\frac{1}{2}\int_{t_0}^{t}\int_{t_0}^{s} b^2(u,X_u)\frac{\partial^2 a(u,X_u)}{\partial x^2}duds + \int_{t_0}^{t}\int_{t_0}^{t} b(u,X_u)\frac{\partial a(u,X_u)}{\partial x}dW_u ds.$$

One can follow that

$$E\left(\mathcal{R}_a(t)|\mathcal{F}_{t_0}\right) = \mathcal{O}\left(\Delta^2\right) \text{ and } E\left(\mathcal{R}_a^2(t)|\mathcal{F}_{t_0}\right) = \mathcal{O}\left(\Delta^3\right)$$

with $\Delta = t - t_0$. Analogously

$$\begin{aligned}
&\int_{t_0}^{t} b(X_s,s)dW_s \\
= &\int_{t_0}^{t}[b(t_0,X_{t_0}) + \int_{t_0}^{s} a(u,X_u)\frac{\partial b(u,X_u)}{\partial t}du + \int_{t_0}^{s} a(u,X_u)\frac{\partial b(u,X_u)}{\partial x}du \\
&+\frac{1}{2}\int_{t_0}^{s} b^2(u,X_u)\frac{\partial^2 b(u,X_u)}{\partial x^2}du + \int_{t_0}^{t} b(u,X_u)\frac{\partial b(u,X_u)}{\partial x}dW_u]dW_s \\
= &\ b(t_0,X_{t_0})\int_{t_0}^{t} dW_s + \mathcal{R}_b(t)
\end{aligned}$$

with

$$E\left(\mathcal{R}_b(t)|\mathcal{F}_{t_0}\right) = \mathcal{O}(\Delta^2) \text{ and } E\left(\mathcal{R}_b^2(t)|\mathcal{F}_{t_0}\right) = \mathcal{O}(\Delta^2).$$

This local error estimation is a smart tool to prove the global approximation error. As in the deterministic case stability and consistency are equivalent to convergence. A detailed discussion of this topic can be found in [Sch03].

3.1.1 Euler scheme

Neglecting the remainders leads to the Euler scheme:

$$X_{n+1} = X_n + a\left(t_n, X_n\right)\Delta_{t_n} + b\left(t_n, X_n\right)\Delta W. \tag{3.3}$$

Hereby $\Delta_{t_n} := t_{n+1} - t_n$ is the stepsize and ΔW a random variable. In the case of the first and second model, $\Delta W = \Delta B \sim N(0, \Delta_{t_n})$ is the increment of a Brownian motion.

Obviously the Euler scheme can be obtained more easily as the discretisation of the stochastic integral

$$X_t = X_{t_0} + \int_{t_0}^{t} a(s,X_s)ds + \int_{t_0}^{t} b(s,X_s)dZ_s.$$

Thus we get

$$X_{n+1} = X_n + a\left(t_n, X_n\right)\Delta_{t_n} + b\left(t_n, X_n\right)\Delta Z \tag{3.4}$$

for any stochastic process Z. Particularly it holds for the fractional Brownian motion, too.

It can be derived from stochastic Taylor expansion that the explicit Euler method has a strong convergence order of 0.5. Again this result only holds for semi-martingales.

We call a numerical integration scheme strongly convergent towards the exact solution Z with strong convergence order γ_1 if

$$\lim_{\Delta_t \to 0} \left| E\left(\left| Z_{t_N} - X_N \right| \left| \mathcal{F}_0 \right.\right) \right| \leq C\Delta_t^{\gamma_1}. \tag{3.5}$$

with $\Delta_t = \max\limits_{t_{i_1},\dots,t_{i_n}} |t_{i+1} - t_i|$. The explicit Euler method has a weak convergence order $\gamma_2 = 1$ for semi-martingales. We say that a numerical integration scheme converges weakly towards the exact solution Z if

$$\lim_{\Delta_t \to 0} \left| E\left(Z_{t_N} | \mathcal{F}_0 \right) - E\left(X_N | \mathcal{F}_0 \right) \right| \leq C\Delta_t^{\gamma_2}. \tag{3.6}$$

3.1.2 Milstein scheme

To obtain the Euler scheme we neglected the remainders. To get an integration scheme of higher order we have to take into account the next higher term of the stochastic Taylor expansion. Therefore we compute one more addend of the remainder term by applying the Ito-formula on the addend $\int_{t_0}^{t}\int_{t_0}^{s} b(u, X_u)\frac{\partial b(u,X_u)}{\partial x} dW_u dW_s$. We obtain

$$\begin{aligned}
&\int_{t_0}^{t}\int_{t_0}^{s} b(u, X_u)\frac{\partial b(u, X_u)}{\partial x} dW_u dW_s \\
= &\int_{t_0}^{t}\int_{t_0}^{s} \Big(b(t_0, X_{t_0})\frac{\partial b(0, X_0)}{\partial x} + \int_{t_0}^{u} \frac{\partial}{\partial t}(b(v, X_v)\frac{\partial b(v, X_v)}{\partial x})dv \\
+ &\int_{t_0}^{u} (a(v, X_v)\frac{\partial}{\partial x} + \frac{1}{2}b^2(v, X_v)\frac{\partial^2}{\partial x^2})(b(v, X_v)\frac{\partial b(v, X_v)}{\partial x})dv
\end{aligned}$$

$$
\begin{aligned}
&+ \int_{t_0}^{u} b(v, X_v) \frac{\partial}{\partial x}\Big(b(v, X_v) \frac{\partial b(v, X_v)}{\partial x} dW_v\Big) dW_u dW_s \\
&= b(t_0, X_{t_0}) \frac{\partial b(t_0, X_{t_0})}{\partial x} \int_{t_0}^{t} \int_{t_0}^{s} dW_u dW_s + \mathcal{R}_{bb}(t)
\end{aligned}
$$

with

$$E\left(\mathcal{R}_{bb}(t) | \mathcal{F}_{t_0}\right) = \mathcal{O}(\Delta^2) \text{ and } E\left(\mathcal{R}_{bb}^2 | \mathcal{F}_{t_0}\right) = \mathcal{O}(\Delta^3).$$

Discretisation and neglecting the remainder terms leads to the explicit Milstein scheme for semi-martingales

$$
\begin{aligned}
X_{n+1} &= X_n + a\left(t_n, X_n\right) \Delta_{t_n} + b\left(t_n, X_n\right) \Delta R_n \\
&+ \frac{1}{2} b\left(t_n, X_n\right) \frac{\partial b}{\partial x}\left(t_n, X_n\right) \left(\left(\Delta R_n\right)^2 - \Delta_{t_n}\right)
\end{aligned}
$$

as

$$\int_{t_0}^{t} \int_{t_0}^{s} dW_u dW_s = \int_{t_o}^{t} W_s dW_s = \frac{1}{2}\left(\left(W_t - W_{t_0}\right)^2 - (t - t_0)\right).$$

Because of its analytical properties we use the implicit Milstein scheme or more precisely the drift implicit Milstein scheme. That means that X_n in the drift term is replaced by X_{n+1}. The implicit Milstein scheme applied on (A.1) is

$$
\begin{aligned}
X_{n+1} = X_n &+ a\left(t_{n+1}, X_{n+1}\right) \Delta_{t_n} + b\left(t_n, X_n\right) \Delta R_n \\
&+ \frac{1}{2} b\left(t_n, X_n\right) \frac{\partial b}{\partial x}\left(t_n, X_n\right) \left(\left(\Delta R_n\right)^2 - \Delta_{t_n}\right).
\end{aligned}
$$

The implicit Milstein possesses a weak convergence order of $\gamma_1 = 1$ and a strong convergence order of $\gamma_2 = 1$. Thus it is more complicated as the explicit Euler but has a higher strong convergence order.

Furthermore it covers one decisive characteristic of the SDEs (2.1) and (2.10) (with appropriate parameter constellation). They are analytically positive. Therefore the integration schemes are supposed to maintain this property.

Definition 3.1 An integration scheme is said to be positive (or to have an eternal lifetime) if

$$P\left(\{X_{n+1} > 0 | X_n > 0\}\right) = 1.$$

The explicit Euler scheme cannot satisfy this demand, but the implicit Milstein can (see [Kah04]). Therefore it is a suitable choice for integrating the SDEs (2.1) and (2.10). Unfortunately the Milstein scheme does not hold for fractional Brownian motion. So far there is only the explicit Euler scheme presented to integrate fractional Brownian motion.

3.2 Integration schemes for fBM

The SDE (2.11) is also supposed to describe a positive process. But the explicit Euler cannot provide positivity. Practically it would be sufficient if almost all trajectories remain positive or if

$$P\left(\{X_{n+1} > 0 | X_n > 0\}\right) \approx 1.$$

Tests show (see chapter 4, 4.11) that the explicit Euler does not fulfill this weaker property either. Integrating (2.11) or (2.17) with the estimated parameters leads to a high percentage of negative trajectories.

As the theory of stochastic Taylor expansion for fractional Brownian motion has not been developed yet one possible approach is focusing on the Euler scheme and trying to improve it.

Remark: There are Ito formulas for fractional Brownian motion but they need derivatives which cannot be computed in general.

3.2.1 Balanced implicit method

One suitable improvement of the straightforward Euler scheme is the balanced implicit method (BIM)

$$\begin{aligned} X_{n+1} &= X_n + a\left(t_n, X_n\right)\Delta_{t_n} + b\left(t_n, X_n\right)\Delta Z + \left(X_n - X_{n+1}\right) C_n\left(X_n\right) \\ C_n\left(X_n\right) &= c_0\left(X_n\right)\Delta + c_1\left(X_n\right)\left|\Delta Z\right| \end{aligned}$$

with bounded control functions c_0 and c_1. It must hold for the control functions that

$$1 + c_0\left(X_n\right) > 0, \tag{3.7}$$

$$c_1\left(X_n\right) \geq 0 \tag{3.8}$$

(see [Sch97a]). It does also hold for processes which do not have the semimartingale property because its derivation does not use the Ito formula.

As the control functions depend on the process, the balanced implicit method cannot guarantee positivity and convergence in general. But for every process (or class of processes) the control functions must be determined newly.

Particularly we want to integrate (2.11) and (2.17). In this case the BIM is not positive but the control functions can be chosen such that it guarantees a weaker kind of positivity.

Definition 3.2 An integration scheme is said to be ϵ-positive for a constant $\epsilon > 0$ if

$$P\left(\{X_{n+1} > 0 | X_n > \epsilon\}\right) = 1.$$

Indeed the BIM is ϵ-positive for (2.11) with $p > 0$ and the control functions

$$\begin{aligned} c_0 &= \kappa, \\ c_1 &= \sigma \max\left(x, \epsilon\right)^{p-1}. \end{aligned}$$

Proof: If $X_n > \epsilon$ the following calculation holds

$$\begin{aligned} X_{n+1} &= \frac{X_n + \kappa\theta\Delta - \kappa\Delta X_n + \sigma X_n^p \Delta B^H + X_n\left(\kappa\Delta + \sigma \max\left(X_n, \epsilon\right)^{p-1} |\Delta B^H|\right)}{1 + c_0\left(X_n\right)\Delta_{t_n} + c_1\left(X_n\right)|\Delta B^H|} \\ &= \frac{X_n + \kappa\theta\Delta + \sigma X_n^p \Delta B^H + \sigma X_n^p |\Delta B^H|}{1 + c_0\left(X_n\right)\Delta_{t_n} + c_1\left(X_n\right)|\Delta B^H|} \\ &> 0. \end{aligned}$$

In case of a deterministic function $\theta(t)$ the BIM preserves positivity if

$$\theta(t_{n+1}) - \theta(t_n) + \kappa\Delta\theta(t_n) \geq 0$$

$$\Longleftrightarrow \frac{\theta(t_{n+1})}{\theta(t_n)} \geq 1 - \kappa\Delta.$$

For the different integration schemes the increments of Brownian motion and fractional Brownian motion are requisite. As the increments of a Brownian motion with stepsize Δ are independently $N(0, \sqrt{\Delta})$ distributed one can use a pseudo random generator in the simulation.
The increments of a fractional Brownian motion are $N(0, \sqrt{\Delta^{2H}})$ distributed but in general not independently. Therefore we cannot simply use the pseudo random numbers. A little more effort is necessary to simulate general Gaussian noise. There are various algorithms among others the Hosking method (see [Hos84]).

3.2.2 Hosking method

Within this thesis the Hosking method is used to simulate fractional Gaussian noise $(X_k)_{k\in\mathbb{N}_0}$ although it can be applied more generally. The presentation bases on [Die02]. The idea of the Hosking algorithm is to determine the distribution of X_{n+1}, if $X_n, ..., X_0$ are given.

The following notations are used: The covariance kernel describes the stochastic relations between the increment X_n and the following. It is given by

$$r(k) = E(X_n X_{n+k}) \tag{3.9}$$

with $n, k \in \mathbb{N}_0$. Without loss of generality $r(0) = 1$ is assumed.
The $(n+1)\times(n+1)$-covariance matrix is given by

$$\Gamma(n) = (r(|i-j|))_{i,j=0,\dots,n}.$$

Furthermore a covariance vector

$$c(n) = (r(1), ..., r(n+1))^T$$

is constructed. With the indicator function χ the $(n+1)\times(n+1)$-matrix

$$F(n) = (\chi_{\{i=n-j\}})_{i,j=0,\dots,n}$$

is a flip matrix.
To simplify the notation the following abbreviations are used

$$\begin{aligned}
d(n) &= \Gamma(n)^{-1}c(n),\\
\tau_n &= d(n)^T F(n) c(n) = c(n)^T F(n) d(n),\\
\Phi_n &= \frac{r(n+2)-\tau_n}{\sigma_n^2}.
\end{aligned}$$

With these notations the matrix Γ can be expressed recursively:

$$\begin{aligned}
\Gamma(n+1) &= \begin{pmatrix} 1 & c(n)^T \\ c(n) & \Gamma(n) \end{pmatrix} \\
&= \begin{pmatrix} \Gamma(n) & F(n)\cdot c(n \\ c(n)^T\cdot F(n) & 1 \end{pmatrix}.
\end{aligned}$$

Calculation shows that for the inverse matrix

$$\Gamma(n+1)^{-1} = \frac{1}{\sigma_n^2}\begin{pmatrix} 1 & -d(n)^T \\ -d(n) & \sigma_n^2\Gamma(n)^{-1} + d(n)d(n)^T \end{pmatrix} \tag{3.10}$$

$$(3.11) \qquad = \frac{1}{\sigma_n^2}\begin{pmatrix} \sigma_n^2\Gamma(n)^{-1} + F(n)d(n)d(n)^T F(n) & -F(n)d(n) \\ -d(n)^T F(n) & 1 \end{pmatrix}$$

holds.

Now we want to prove that

$$X_{n+1} \sim N(\mu_n, \sigma_n^2)$$

with

$$\begin{aligned} \mu_n &= c(n)^T\Gamma(n)^{-1}(X_n, ..., X_0)^T, \\ \sigma_n^2 &= 1 - c(n)^T\Gamma(n)^{-1}c(n). \end{aligned}$$

It is only necessary to put the pieces together. One can conclude from (3.10) that

$$(y, x^T)\Gamma(n+1)^{-1}\begin{pmatrix} y \\ x \end{pmatrix} = \frac{(y - d(n)^T x)^2}{\sigma_n^2} + x^T\Gamma(n)^{-1}x$$

holds for $x \in \mathbb{R}^{n+1}$ and $y \in \mathbb{R}$ which shows that $X_{n+1} \sim N(\mu_n, \sigma_n^2)$ indeed.

Now it is possible to determine the distribution of X_{n+1} if $X_0, ..., X_n$ are known (respectively have been already simulated). This theoretical result becomes usable by constructing a recursion for μ_n and σ_n. Again with (3.10) computing shows that

$$\begin{aligned} \sigma_{n+1}^2 &= \sigma_n^2 - \tfrac{(\gamma(n+2)-\tau_n)^2}{\sigma_n^2}, \\ d(n+1) &= \begin{pmatrix} d(n) - \Phi_n F(n)d(n) \\ \Phi_n \end{pmatrix} \end{aligned}$$

is valid which leads explicitly to σ_n and implicitly to μ_n.
The recursion starts with $\mu_0 = r(1)X_0, \sigma_0^2 = 1 - r(1)^2$ and $\tau_0 = r(1)^2$.

To sum up the Hosking algorithm allows to simulate fractional Gaussian noise. Thus the numerical integration for model 3 and 4 can be implemented as well.

Summary

As the models set up in chapter 2 cannot be analytically integrated in general numerical integration schemes are requisite. The Euler scheme was presented. It is the discretisation of a stochastic integral equation. This scheme allows to integrate all four models but has a low convergence order and furthermore leads to negative integration paths. That is undesirable as it does not

correspond with the analytical properties of the SDEs respectively does not correspond with the properties of rain which are described by these SDEs.
In case of ordinary Brownian motion we can use the Ito formula which is the essential tool to set up stochastic Taylor expansion. The Euler scheme for Brownian motion can be derived from stochastic Taylor expansion by neglecting all terms of higher order than 1. Taking into account the next higher term the Milstein scheme is obtained. The implicit Milstein scheme is an appropriate tool to integreate (2.1) and (2.10) as it has a higher strong convergence order as the Euler scheme and as it can preserve positivity.
To integrate the SDEs driven by fractional Brownian motion ((2.11) and (2.17)) positively an extension of the Euler scheme was presented, the Balanced implicit method.
Eventually in case of fractional Brownian motion correlated random numbers are necessary for numerical integration. The Hosking algorithm which creates them was presented.

CHAPTER IV

Numerical tests and validation

The chapter is divided into two sections. Firstly the results of parameter estimation are presented. Secondly prices are calculated. Mostly the weather station Schleswig services as example. Data from 1947 till 2003 are used. Months are the underlying time unit and 0.1 mm per m^2 the basic rain unit. Thus if $\hat{\theta} = 740$ is estimated it means that there was an average precipitation of 74 mm per m^2 at the weather station Schleswig during the years 1947 till 2003.

4.1 Parameters

Within this subsection the stability of the different parameter estimators is analysed. It mainly focus on the impact of the length of data series and on how far the parameters respectively their estimators influence each other.

4.1.1 Model 1

Drift parameters

The theoretical results in chapter 2 prove that with the realisations $Y_1, Y_2, \dots$ the estimators

$$\hat{\theta} = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

for θ and

$$\hat{\kappa} = \frac{1}{n} \sum_{i=1}^{n} \frac{Y_{i+1} - Y_i}{\theta - Y_i}$$

for κ are unbiased.
Figures 4.1 and 4.2 underline that this property holds in praxis, too. For figure 4.1 one path is simulated with the given parameters θ, κ, σ and p. From this simulated path θ is reestimated. In figure 4.2 the unknown θ is estimated from real historic data series. Hereby the length of data taken into account increases. One observes that the path of estimated θ seems to converge towards $\hat{\theta} \approx 740$.

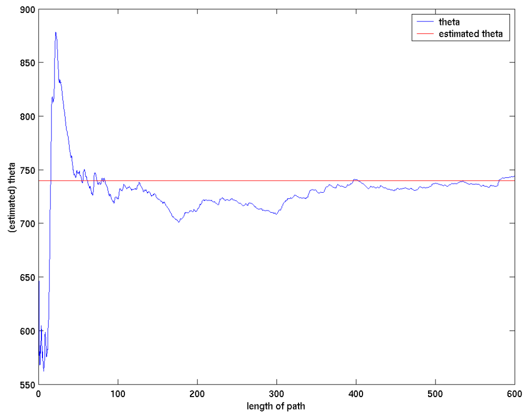

Figure 4.1: $\hat{\theta}$ vs. length (simulated) data series
$\kappa = 1.1, \sigma = 6.7, p = 1.0, h = 0.25, \theta = 739.8$

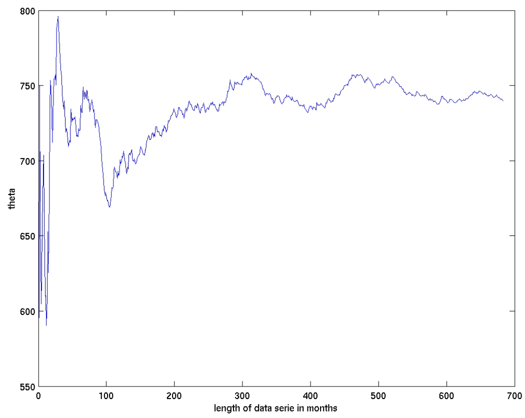

Figure 4.2: θ vs. length (real) data series
Schleswig, data series: 1947-2003

Now the modified κ-estimator

$$\hat{\kappa_b} = \frac{1}{\#I_b} \sum_{i \in I_b} \frac{X_{i+1} - X_i}{(\theta(i) - X_i)\,\Delta}.$$

is considered. To test the modification on suitability the estimator is plotted against the boundary b (as in chapter 2, figure 2.5). Again a path is simulated with the given parameters θ, κ, σ and p. Afterwards κ is reestimated depending on b.

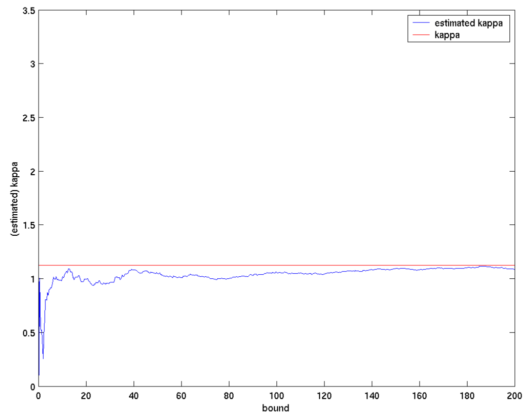

Figure 4.3: κ vs. bound
$\theta = 739.8, \sigma = 6.7, p = 1.0, h = 0.25$

One sees that the unmodified estimator (thus $b = 0$) is far away from the solution κ. With growing b the estimation draws near the real solution. Even for a very high boundary of 100 the modified estimator is much better than the unmodified. Thus the modified estimator proves to be stable concerning the bound.
What the picture does not show is the behaviour for even higher bounds. It does not become unstable but it simply stops. Depending on the path there is a bound which is so high that no data points are taken into account anymore. So after a particular bound the estimation does not work anymore, but as long as it works it is stable.
In the following the modified estimator $\hat{\kappa_b}$ with $b = 20$ is used.

As in the case of θ in figure 4.4 $\hat{\kappa_b}$ is plotted against the length of data series.

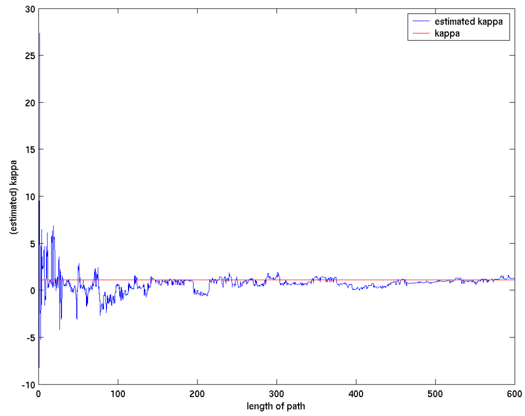

Figure 4.4: κ_b vs. length (simulated) data series
$\theta = 739.8, \sigma = 6.7, p = 1.0, h = 0.25$

Although in chapter 2 there is only proven that $\hat{\kappa}$ is unbiased $\hat{\kappa_b}$ seems to be unbiased, too.

Figure 4.5 shows the impact of θ on the estimation of κ. The path is simulated with $\theta = 739.8$.

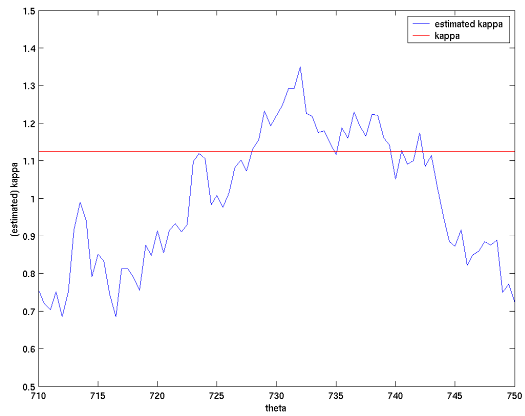

Figure 4.5: Impact of θ on κ
$\theta = 739.8, \kappa = 1.1, \sigma = 0.667, p = 0.981, h = 0.25$

As expected θ strongly influences $\hat{\kappa_b}$. But with rather exact estimation of θ one gets sufficient results.

Diffusion parameters

The diffusion parameters σ and p are estimated in one step. They are calculated from a smoothing function. In figure 4.6 only 15 points can be seen because the data points have been accumulated in buckets. By doing so the influence of the statistical mavericks is decreased.

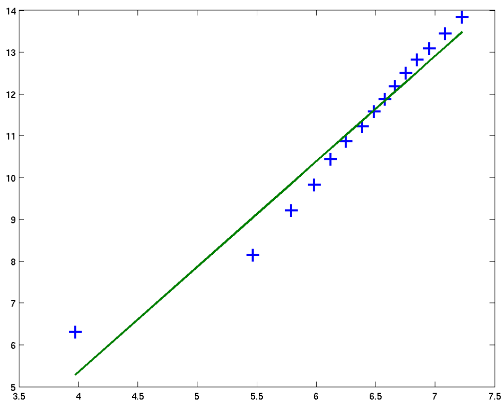

Figure 4.6: Best fit straight line
for estimating σ and p

Again the relation between path length and estimation is analysed (see figures 4.7 and 4.8).

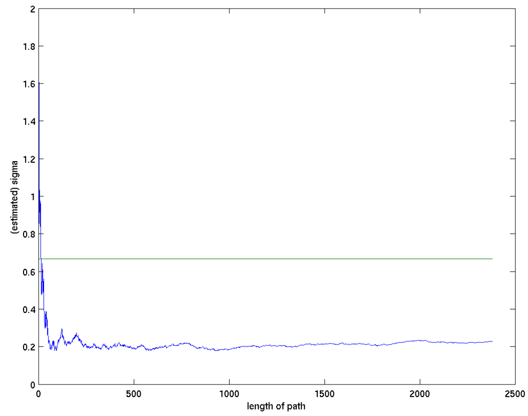

Figure 4.7: σ vs. length (simulated) data series
$\theta = 739.8, \kappa = 1.1, h = 0.25$

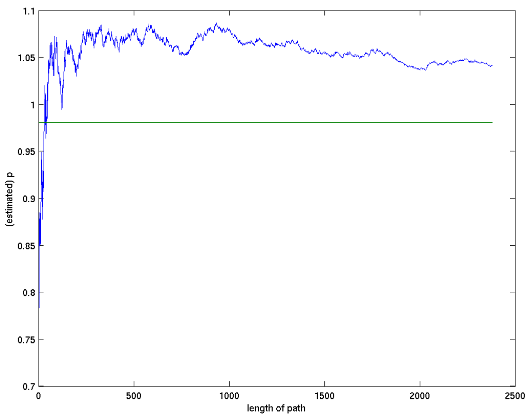

Figure 4.8: p vs. length (simulated) data series
$\theta = 739.8, \kappa = 1.1, h = 0.25$

Also in this case the estimators stabilise with increasing path length. But it is remarkable that σ is estimated below the real value and p above. This effect appears for other paths as well. A possible explanation is that this effect is due to the used integration scheme.

To test this assumption the same procedure as above is made with a path simulated by the BIM. The result is visualised in the following figures 4.9 and 4.10:

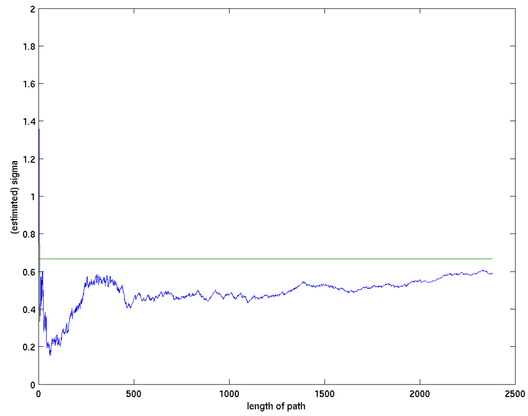

Figure 4.9: σ vs. length (simulated) data series, BIM
$\theta = 739.8, \kappa = 1.1, h = 0.25$

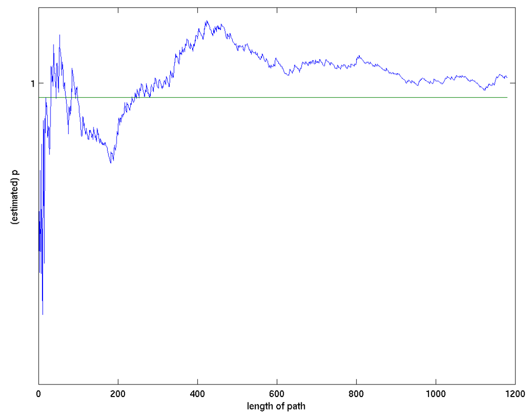

Figure 4.10: p vs. length (simulated) data series, BIM
$\theta = 739.8, \kappa = 1.1, h = 0.25$

Firstly the lower convergence speed of the BIM is reflected in the higher fluctuations. But in contrast to the Milstein scheme the estimators of p and σ slowly converge against the input values.
Obviously the integration schemes differently deal with the volatility parameters. (There is no decisive difference considering the drift parameters.) As this property is most likely to influence the pricing only one integration

scheme is used if prices are compared. There are two reasons for the BIM: On the hand there is a convergence. On the other hand only the BIM can handle Brownian and fractional Brownian motion. Thus the BIM is chosen in those cases.

4.1.2 Model 2

As the estimation of the function $\theta(t)$ is new only the estimation of its parameters is analysed in this subsection. Concerning κ, σ and p the results from above hold analogously.

In chapter 2 the presented plots show $\theta(t)$ with 2 sine terms. The approximation to the real averages seems sufficiently. More general the function

$$\theta(t) = m + \sum_{i=0}^{n} \alpha_i \sin\left(\frac{(2i+1)\,\pi\,(t-v)}{6}\right)$$

was proposed. See remark 4.1 why $n = 3$ is an appropriate choice.

The parameters $m, \alpha_0, \alpha_1, \alpha_2, \alpha_3$ and v are estimated with regard to minimising the difference to the average (see also 2). Thus the estimators for $m, \alpha_0, \alpha_1, \alpha_2, \alpha_3$ and v are obtained as minimiser of

$$\min_{m,\alpha_0,\alpha_1,\alpha_2,\alpha_3,v} \| f(\cdot, m, \alpha_0, ..., \alpha_3, v) - D(\cdot) \|_2^2$$

with

$$\begin{aligned} f(t, m, \alpha_0, \alpha_1, \alpha_2, \alpha_3, v) = m &+ \alpha_0 \sin\left(\frac{\pi\,(t-v)}{6}\right) + \alpha_1 \sin\left(\frac{3\pi\,(t-v)}{6}\right) \\ &+ \alpha_2 \sin\left(\frac{5\pi\,(t-v)}{6}\right) + \alpha_3 \sin\left(\frac{7\pi\,(t-v)}{6}\right) \end{aligned}$$

and $D(t)$ average at t. There are a lot of numerical schemes to solve this least-square problem. For this thesis the Gauss-Newton algorithm is used (see for example [Sch97b]).

A slightly different procedure is estimating m as $\hat{\theta}$ in advance (as in model 1) because the process is still supposed to fluctuate around that mean. Then one computes the minimiser of

$$\min_{\alpha_0,\alpha_1,\alpha_2,\alpha_3,v} \| \hat{f}(\cdot, \alpha_0, ..., \alpha_3, v) - D(\cdot) \|_2^2$$

with

$$\begin{aligned}\hat{f}(t, \alpha_0, \alpha_1, \alpha_2, \alpha_3, v) = \hat{\theta} \quad &+ \quad \alpha_0 \sin\left(\frac{\pi\,(t-v)}{6}\right) + \alpha_1 \sin\left(\frac{3\pi\,(t-v)}{6}\right) \\ &+ \quad \alpha_2 \sin\left(\frac{5\pi\,(t-v)}{6\pi}\right) + \alpha_3 \sin\left(\frac{7\pi\,(t-v)}{6\pi}\right).\end{aligned}$$

It does lead to only marginally different values e. g. 736.9 becomes 739.8 in the second estimation.

Remark 4.1 There is still the question to answer how many sine terms are necessary. The solution can only be found individually. Our example is the weather station Schleswig but there similar results for the other weather stations mentioned in chapter 1.
Considering 1, 2, 3 or 4 sine terms leads to the following parameter constellations:

	m	α_0	α_1	α_2	α_3	v
$n=1$	736.9	191	-	-	-	0.485
$n=2$	736.9	191	39	-	-	0.479
$n=3$	736.9	191	39	-14	-	0.479
$n=3$	736.9	191	39	74	-61	0.479

Table 4.1: Impact of number of sine terms

Although there seems to be a significant change from three to four sine terms, the effect on the pricing is very small (see table 4.7). This also holds for the step from 2 to 3 and even for the step from 1 to 2 sine terms. Therefore the choice of 4 sines is maybe not necessary but anyway sufficient. Thus the function

$$\theta(t) = m + \sum_{i=0}^{3} \alpha_i \sin\left(\frac{(2i+1)\,\pi\,(t-v)}{6}\right)$$

is considered in this (and also in the following) chapter.

4.1.3 Model 3

Nothing new comes up concerning the drift parameters. But the diffusion parameters now include the Hurst parameter.
Two different possibilities to estimate the diffusion parameters are introduced in chapter 2. They depend on the stepsize. The results of both methods are

compared. As an example stepsizes of $\Delta = 1$ respectively $\Delta = 0.25$ are considered:

	$\Delta = 1$	$\Delta = 0.25$
σ	0.667	0.600
p	0.981	0.996
H	0.535	0.531

Table 4.2: Estimation of diffusion parameters in model 3
$\theta = 739.8, \kappa = 1.251$

One observes that estimations differ. There are no big differences in estimation p and H. But the variation of σ-calibration is stronger. In the next subsection the influence of these estimation triples on the prices is analysed (see table 4.9).

As there is no new estimation technique in model 4 no further analysis is needed. The logically next step is the initial motivation - the pricing of options.

4.2 Integration

Four different options are considered in this section, they refer to a rain index summing up rain

$$R_{S,T} = \sum_{i=S}^{T} X_i.$$

The payoff functions of the options are as follows

- Call: $f(X, S, T) = (R_{S,T} - K)^+$, $K = 11096$;
- Put: $f(X, S, T) = (K - R_{S,T})^+$, $K = 7989$;
- Barrier: $f(X, S, T) = (R_{S,T} - K)^+ \cdot 1_{\{R_{S,T} > B\}}$, $K = 11096$, $B = 12428$;
- Binary: $f(X, S, T) = 1000 \cot 1_{\{\max\limits_{S \leq i \leq T} X_i > B\}}$, $B = 1997$.

Note that they are all Asian type option.
The parameter constellations base on the first section of this chapter.

4.2.1 Euler versus Milstein and BIM

The interest of this thesis is not analysing numerical integration schemes with regard to their numerical properties like convergence. But it is necessary to find integration schemes which map the analytical properties of the models (2.1), (2.10), (2.11) and (2.17). As already mentioned all these models are supposed to describe positive stochastic processes. One can theoretically show that the Euler method cannot preserve positivity (see [Kah04]). The following pictures are only supposed to underline this theoretical result. Furthermore one gets an impression how many paths become negative depending on stepsize and path length.

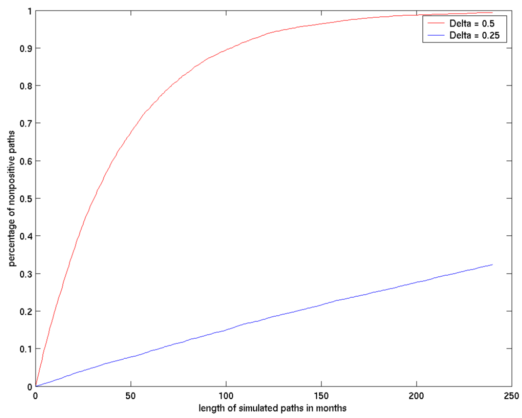

Figure 4.11: Percentage of negative paths (out of 10000 paths)
$\theta = 739.8, \kappa = 1.125, \sigma = 0.667, p = 0.981, T = 12$ months

Figure 4.12 shows one path simulated with the Euler scheme. It becomes visible that this integration scheme creates trajectories which are not similar to real rain paths.

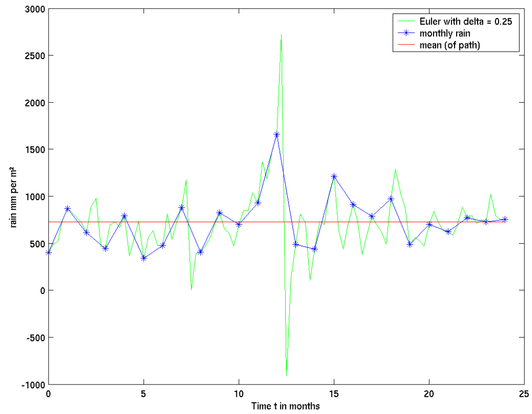

Figure 4.12: One path simulated with the Euler method
$\theta = 739.8, \kappa = 1.125, \sigma = 0.667, p = 0.981, \Delta = 0.25$

As a comparison one path simulated with the Milstein scheme and one simulated with the BIM are presented.

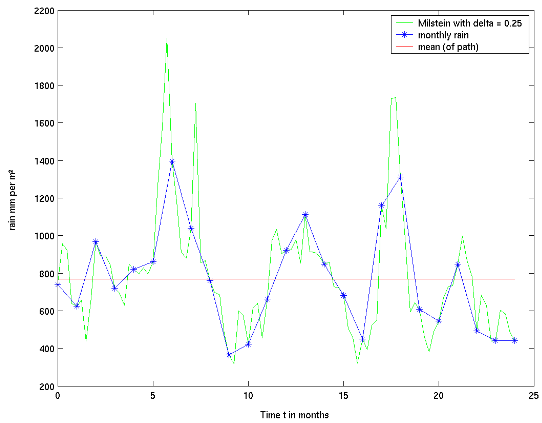

Figure 4.13: One path simulated with the Milstein method
$\theta = 739.8, \kappa = 1.125, \sigma = 0.667, p = 0.981, \Delta = 0.25$

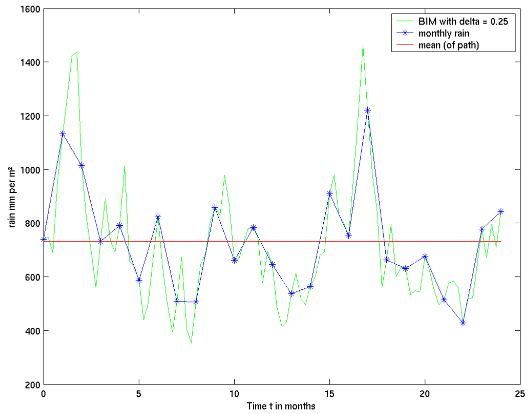

Figure 4.14: One path simulated with the BIM
$\theta = 739.8, \kappa = 1.125, \sigma = 0.667, p = 0.981, \Delta = 0.25$

The Euler method applied on (2.10), (2.11) and (2.17) does not work better than in the case of (2.1). Therefore the Milstein scheme or the Balanced Implicit method is used in the following.

4.2.2 Model 1

The four options are priced assuming that rain follows the SDE (2.1). The integration happens with the BIM. The prices are calculated with Monte Carlo-simulation. The first picture shows that Monte-Carlo-simulation is a suitable pricing tool if the number of simulated paths is sufficiently high:

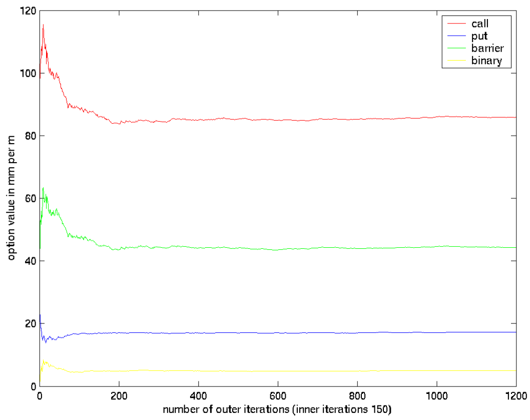

Figure 4.15: Convergence BIM in model 1
$\theta = 739.8, \kappa = 1.125, \sigma = 0.667, p = 0.981, \Delta = 0.25, T = 12$ months

One sees that the put converges faster than the other options.

The prices clearly depend on the parameter input. So this impact is analysed in the following.
Firstly θ is dealt with. The other parameters remain fix as $\kappa = 1.125, \sigma = 0.667$ and $p = 0.981$. Again the stepsize is 0.25 and the maturity 12 months. Values between 735 and 755 are considered reflecting the variation in estimation.

	Call	Put	Barrier	Binary
$\theta = 735$	77	20	39	5
$\theta = 740$	85	17	44	5
$\theta = 745$	94	15	49	6
$\theta = 750$	101	13	53	6
$\theta = 755$	109	11	57	6

Table 4.3: Impact of θ on option prices, BIM
$\kappa = 1.125, \sigma = 0.667, p = 0.981, \Delta = 0.25, T = 12$ months

As expected one sees that call, barrier and binary prices increase with increasing θ. Vice versa the put value goes down.

In table different κ and their influence on the prices are compared.

	Call	Put	Barrier	Binary
$\kappa = 0.95$	129	34	80	11
$\kappa = 1.00$	117	27	70	9
$\kappa = 1.05$	103	23	58	7
$\kappa = 1.10$	89	19	46	6
$\kappa = 1.15$	80	16	40	5
$\kappa = 1.20$	70	13	31	4

Table 4.4: Impact of κ on option prices, BIM
$\theta = 739.8, \sigma = 0.667, p = 0.981, \Delta = 0.25, T = 12$ months

Obviously κ has strong influence on the prices. The lower the κ is the higher the call and barrier prices are. That is plausible because a low speed of mean reversion forces the stochastic process to return from extremes slowly. Thus the phases of very high and of very low prices are long. As the call option possesses an asymmetric risk structure, it superproportionally profits from the ups which leads to a higher option price.
A similar argument holds for the binary option. Long up phases lead to higher probability of a high maximum. Thus it decreases with growing κ too.
The put option also increases as it profits from the downs above average.

Most option pricing models strongly react on the input of the volatility parameters. Here it holds, too.

	Call	Put	Barrier	Binary
$\sigma = 0.55$	41	7	14	3
$\sigma = 0.60$	55	11	23	5
$\sigma = 0.65$	71	17	34	10
$\sigma = 0.70$	87	23	45	14

Table 4.5: Impact of σ on option prices, BIM
$\theta = 739.8, \kappa = 1.125, p = 0.981, \Delta = 0.25, T = 12$ months

Clearly a high σ leads to high option prices. Again the reason for this behaviour is the asymmetric risk structure of options. A high volatility leads to high probability for fortunate sceneries and to a high probability of unfortunate sceneries. Because of the asymmetric risk structure an option holder can profit from this fortunate scenery but he does not have to carry the risk of the unfortunate sceneries. This leads to heavily increasing prices if σ increases.

The next table implies that the second volatility parameter p has a similar impact on the option prices, as a bigger p leads to higher volatility.

	Call	Put	Barrier	Binary
$p = 0.94$	30	5	8	2
$p = 0.96$	48	10	18	4
$p = 0.98$	75	19	37	10
$p = 1.00$	108	31	63	23

Table 4.6: Impact of p on option prices, BIM
$\theta = 739.8, \kappa = 1.125, \sigma = 0.667, \Delta = 0.25, T = 12$ months

To sum up the parameter estimation proves to be a very sensitive point in pricing options. Especially the estimation of the diffusion parameters should be done very carefully.

4.2.3 Model 2

It follows from the parameter estimation that

$$\begin{aligned}\theta(t) = 739.8 \quad &+ \quad 191 \sin\left(\frac{\pi\,(t-0.479)}{6}\right) + 39 \sin\left(\frac{3\pi\,(t-0.479)}{6}\right)\\ &+ \quad 74 \sin\left(\frac{5\pi\,(t-0.479)}{6}\right) - 16 \sin\left(\frac{7\pi\,(t-0.479)}{6}\right)\end{aligned}$$

and

$$\kappa = 1.123.$$

Calculation shows that

$$\frac{\theta(t_{n+1})}{\theta(t_n)} \geq 1 - \kappa\Delta$$

holds for $\Delta = 0.25$ which is the stepsize used in the following. Thus the BIM integrates (2.10) positively in this situation.

As for model 1 the relation between path length and prices is visualised.

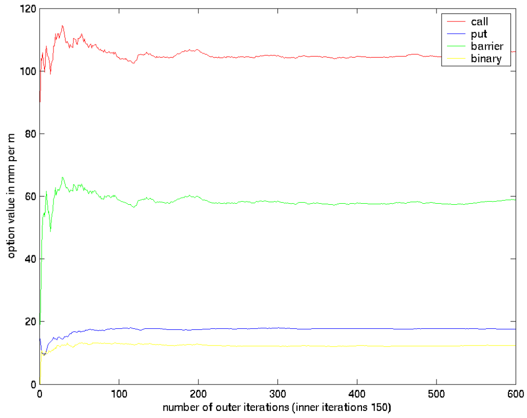

Figure 4.16: Convergence BIM in model 2
$\theta = 739.8, \alpha_0 = 191, \alpha_1 = 39, \alpha_2 = 74, \alpha_3 = -16, v = 0.479, \kappa = 1.123, \sigma = 0.667, p = 0.981, \Delta = 0.25, T = 12$ months

Again one can conclude that a Monte Carlo-simulation with more than $50,000$ paths leads to stable option prices.

As already announced the impact of the number of sine terms on the prices is analysed. Hereby $m = 739.8$ and number of paths $n = 180.000$.

	Call	Put	Barrier	Binary
$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0, v = 0.49$	89	18	46	6
$\alpha_1 = 191, \alpha_2 = \alpha_3 = \alpha_4 = 0, v = 0.49$	100	18	55	12
$\alpha_1 = 191, \alpha_2 = 39, \alpha_3 = \alpha_4 = 0, v = 0.48$	106	18	58	12
$\alpha_1 = 191, \alpha_2 = 39, \alpha_3 = -14, \alpha_4 = 0, v = 0.48$	104	18	56	12
$\alpha_1 = 191, \alpha_2 = 39, \alpha_3 = 74, \alpha_4 = -16, v = 0.48$	106	18	58	12

Table 4.7: Impact of number of sine terms on option prices, BIM
$\theta = 739.8, \kappa = 1.123, \sigma = 0.667, p = 0.981, \Delta = 0.25, T = 12$ months

Table 4.7 shows that there is a significant difference whether using one sine term or a constant mean. But the next sine terms have only little influence. Thus practically one or two sine terms would probably serve very well. To cover a wider range of possible scenarios four sine addends are used in the following.

Comparing model 1 and model 2 one sees that the sine-mean leads to higher or stable option prices. Particularly the Binary option is much more expensive if priced with model 2. Its value is doubled. That is plausible as the random process fluctuates around a non deterministic mean. Thus at the high points of the sine curve the probability to go beyond the barrier is higher as if there were a constant mean.

4.2.4 Model 3

It is interesting to see if the Balanced Implicit method leads to convergent prices also in this case.

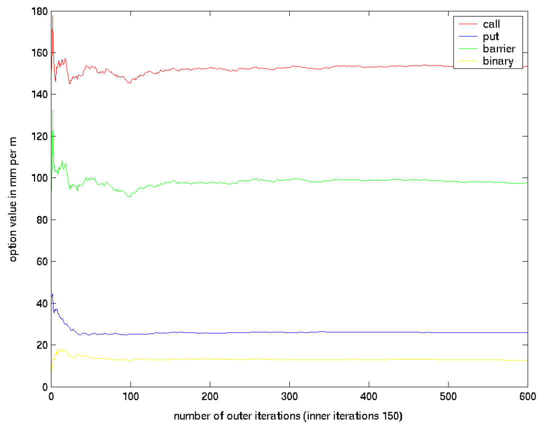

Figure 4.17: Convergence BIM in model 3
$\theta = 739.8, \kappa = 1.125, \sigma = 0.667, p = 0.981, H = 0.535, \Delta = 0.25, T = 12$ months

Actually $n = 50,000$ paths delivers stable option prices.

As there is a new parameter - the Hurst factor - the impact of this parameter on the option prices is analysed. The Monte Carlo simulation is done with the BIM.

	Call	Put	Barrier	Binary
$H = 0.50$	83	18	41	5
$H = 0.51$	100	20	53	7
$H = 0.52$	121	22	71	9
$H = 0.53$	144	24	89	11
$H = 0.54$	161	26	105	14

Table 4.8: Impact of the Hurst factor on option prices
$\theta = 740, \kappa = 1.12, \sigma = 0.667, p = 0.981, \Delta = 0.25, T = 12$ months,
$n = 60.000$

One observes that the higher the Hurst factor is the higher the option prices are. Again this is plausible because of the asymmetric risk structure of options. Similar to κ with growing Hurst factor the probability for long up phases and long down phases increases which leads to rising option prices. As in the case of the two other diffusion parameters the influence is strong.

The estimation of σ, p and H depends on the stepsize. Two different triples are presented in the previous section (see table 4.2). Table 4.9 shows their impact on the prices.

	Call	Put	Barrier	Binary
$\sigma = 0.667, p = 0.981, H = 0.535$	150	26	93	12
$\sigma = 0.600, p = 0.996, H = 0.531$	146	25	91	12

Table 4.9: Different diffusion parameters constellations
$\theta = 740, \kappa = 1.125, \Delta = 0.25, T = 12$ months, $n = 60.000$

One observes that the two parameter constellations do not lead to significantly different option prices. Thus from a pricing point of view the triples are equivalent.

4.2.5 Model 4

Again the Monte Carlo-simulation leads to convergent option prices.

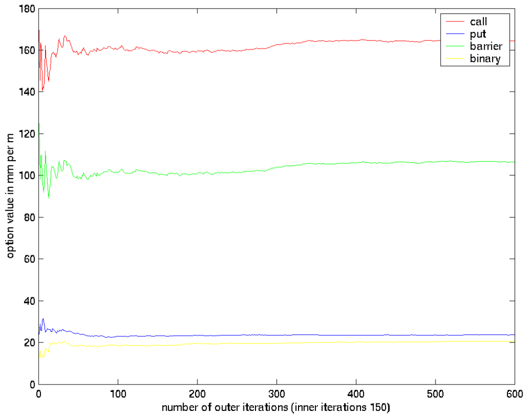

Figure 4.18: Convergence BIM in model 4
$\theta = 739.8, \alpha_0 = 191, \alpha_1 = 39, \alpha_2 = 73, \alpha_3 = -16, v = 0.479, \kappa = 1.123, \sigma = 0.667, p = 0.981, H = 0.527, \Delta = 0.25, T = 12$ months

Compared to model 2 one observes that the Hurst factor again acts like a price booster. The grade of increasing is similar to the one between model 1 and 3.
A comparison between model 3 and 4 shows that the a constant mean leads to lower option costs. But the difference is not as big as in case of model 1 and 2. That is plausible because the estimator of the Hurst factor in model 4 is lower than in model 3 (0.535 respectively 0.527). That weakens the price boosting effect of the deterministic mean.

Summary

The parameter estimators were tested. As the theoretical results already indicated $\hat{\theta}$ is an unbiased estimator. Further $\hat{\kappa}_b$ turned out to be unbiased. The estimators of the diffusion parameters proved to work as well.
The data series has to be sufficiently long; hereby the drift parameters estimators converge faster.
Considering the diffusion parameters the problem arises that the Milstein scheme does not simulate the given SDEs properly. The diffusion parameters cannot be reestimated from a simulated path.
It turned out that the two presented estimation techniques for H lead to slightly different estimations but to equivalent results concerning the prices.

The estimated Hurst factor is above 0.5, so the rain process is indeed driven by a fractional Brownian motion. An estimation near 0.5 would have meant that the refinement is not necessary as that would mean that the process is driven by an ordinary Brownian motion.

CHAPTER V

Case studies

In the previous chapter rain derivatives are priced based on the models developed in chapter 1. In this chapter the possible impact of rain options on a company's profits and losses is demonstrated. The pricing of the options bases on model 4 with 4 sine terms.

As appropriate real company data were not available a fictitious situation is assumed. Importance is attached to a realistic construction. Therefore public available data flows into the case scenarios so far as they are on hand. The case study deals with two companies strongly exposed to rain risk. The one profits from much rain, the other one from few rain. The study focus on the developments of the two companies over a period of one calendar year.

There are two different rain scenarios analysed. These are the real precipitation distributions from the years 1991 and 2000 at the weather station Düsseldorf.

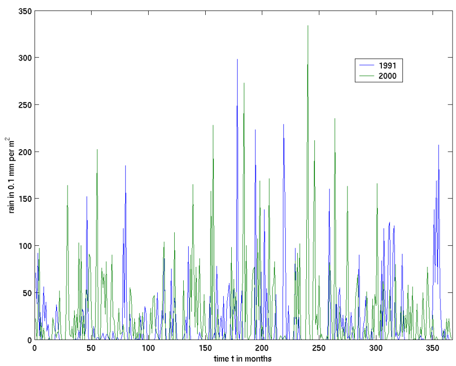

Figure 5.1: Rain in 1991 and 2000, Düsseldorf

On the other hand there are three risk scenarios analysed. We assume that both companies hedge their open rain specific risk exposure either with

- an insurance (Scenario B) or

- a rain derivative (Scenario C) or

- not at all (Scenario A).

To analyse the impact of the three different actions in the two different rain scenarios the profit and loss accounts for both companies are set up.

The companies chosen within this case study are a dam company and a leisure park which are strongly influenced by the weather event precipitation. In this fictitious comparison this influencing factor is focussed on.
To make the companies comparable they are both assumed to be situated near Düsseldorf.
The used rain data are publicly available on www.dwd.de. The years 1969 till 2003 are taken into account for the parameter estimation. The data series are complete, so there are no lacking days.

Rather ordinary put and call options are considered because of two reasons. On the one hand derivatives are supposed to have a pretty high turnover at the equity markets. This is only possible if there is a particular degree of standardisation. On the other hand this circumstances are not yet given. Actually there are no standardised rain products in Germany.
Let us consider one possible situation to show why this fact also argues for simple puts and calls. A company wants to buy a rain derivative to hedge its risk. The most natural and maybe even only available partner is a bank. We assume that the bank sells an option. So now the bank has got an open risk exposure. As trades are not high more likely not existing at the equity markets they cannot resell or hedge their own risk. It would be much more comfortable for the bank if they can find two companies with opposed risk structures and arrange the contract. So that the companies hedge one another (unknowingly) and the bank gets a particular margin without any risk or at least with minimal risk.

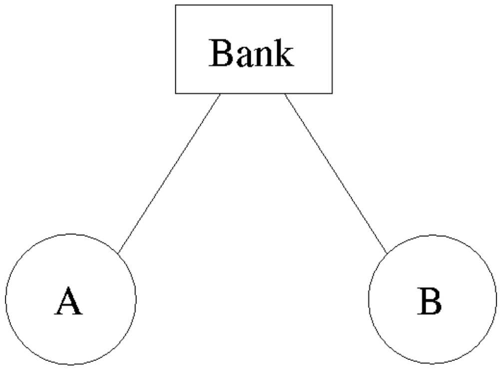

Figure 5.2: Bank as intermediary

The policies are priced with a method called burn analysis (see e.g [Nel96]). It means that one considers an appropriate number of past years and computes the payoff which would have happened in each year. Taking the average leads to the estimated price. Disadvantage is that each year only provides one data set. A low premium of 1% is assumed.

Both security instruments are compared by putting up the P&L of the two companies. The possible payout of the rain options is listed under other operating income. This is justified by the factual right to choose (faktisches Wahlrecht) which holds in the case of weather derivatives.

5.1 The leisure park

The leisure park is publicly open from April to October. In this period there are 179 business days. The main revenues happen in July and August. Additionally some smaller winter events happen. All in all the average revenues are 4.4 million Euro. The company employs 14 permanent employees with a salary of 48,000 Euro gross. In the open months the staff is completed by seasonal workers with a salary of 18,000 (between April and November). Information from www.grevinetcie.com are used.

Insurance

In the insurance scenario the leisure park company buys an insurance with the following features. The contract only protects the period from April till

October. The insurance company has to pay off if the rain in one month increases over a specific barrier. They payoff does not depend on the specific amount of rain but only on the fact if the barrier is reached. For every month with more than 100 mm per m^2 the leisure park gets $310,000$ Euro.
The advantage for the insurance company is that its loss is capped. The advantage for the insurance holder - the leisure park company - is that in most cases they can balance their losses in the operating business. Only in really extreme weather years the payoff does not suffice.

Option

The alternative rain derivative is as already mentioned rather simple. The leisure park company buys a call covering the period April, May, June, July, August, September, October. The call refers to a rain index which sums up the actual precipitation. The strike is $K = 464.8$ mm per m^2 and the tick size $s = 3,800$. That means that the leisure park company gets $3,800$ Euro per every 0.1 mm it rains more than $464,8$.

Obviously this choice does not take into account that there are more and less important months for the companies' revenues. The reasons for this decision are the same reasons which hold for the choice of simple options. It makes the option less special and thereby more tradable.

It is positive for the option seller that his payoff increases proportionally. So in most of the cases he will not have to pay such a big amount of money as in the case of the insurance policy. Furthermore there are the advantages which are usually associated which options. The option may fit with his own risk exposure in so far that the contract means risk hedging for himself (which is rather unlikely in the case of an insurance). On the other hand he may be able to resell the option.
Advantage for the option holder is that the payoff function fits better to its probable profit and loss structure. As well for the holder there is an derivative specific advantage. Probably the derivative is cheaper as the insurance. There are various reasons for this phenomenon. Firstly there are the advantages for the seller (s. a.). Secondly if there is a market for derivatives there is also market price for derivatives. Therefore the margin cannot be determined by the seller. Admittedly an insurance company is also exposed to competition. But the market price building is not that transparent and efficient.

5.1.1 Scenario A

Clearly in this scenario the company unrestrictedly profits from advantageous weather as it has been in 1991. It earns a high profit for the year of 451, 900 Euro which corresponds with 8.8% of the revenues. On the other hand there is no protection against possible negative impact. This reflects in a negative result in 2000. There is a total loss of 82, 600 Euro respectively 2.0% of the revenues.

	Scenario A	1991			2000	
		,	%		,	%
	Revenues	5.120,0	100,0%		4074,6	100,0%
A.	Total operating performance	5.120,0	100,0%	A.	4074,6	100,0%
B.	Cost of goods sold	1.385,4	27,1%	B.	1155,4	28,4%
C.	Gross margin	3.734,6	72,9%	C.	2919,2	71,6%
D.	Personnel expenses	904,2	17,7%	D.	904,2	22,2%
	other operating expenses	1.121,5	21,9%		1116,3	27,4%
	other operating income	419,4	8,2%		419,4	10,3%
E.	EBITDA	2.128,3	41,6%	E.	1318,1	32,3%
F.	Depreciation and amortisation	197,1	3,9%	F.	195,5	4,8%
G.	EBIT	1.931,1	37,7%	G.	1122,6	27,6%
H.	Financial results	-1.202,2	-23,5%	H.	-1205,2	-29,6%
I.	Income from operations	728,9	14,2%	I.	-82,6	-2,0%
J.	Extraordinary results	0,0	0,0%	J.	0,0	0,0%
K.	Earnings before taxes (EBT)	728,9	14,2%	K.	-82,6	-2,0%
L.	Net income / Net loss	451,9	8,8%	L.	-82,6	-2,0%
M.	EAT	451,9	8,8%	M.	-82,6	-2,0%

Figure 5.3: P&L for the leisure park in 1991 and 2000

Although there is a profit per year in total this kind of fluctuating is disadvantageous for the company. It has to provide a lot of equity to fill possible gaps respectively it has to raise expensive credits. In the worst case if it does not get credits to get over the liquidity shortage the company must declare insolvency.

To avoid those negative impacts the company could protect itself against rain risk for example by an insurance. That leads to Scenario B.

5.1.2 Scenario B

The main differences to scenario A are highlighted in red. For clarity a detailed P&L has been omitted at this point. It can be found in the appendix B.

	Scenario B	1991			2000	
		,	%		,	%
	other operating expenses	1.417,1	27,7%		1411,9	34,7%
	other operating income	419,4	8,2%		1039,4	25,5%
M.	EAT	268,7	5,2%	M.	149,9	3,7%

Figure 5.4: P&L cutout for the leisure park in 1991 and 2000, scenario B

The insurance product serves the main purpose very well. Even in the disadvantageous year 2000 there is still a profit for the year of $149,900$. This positive result is reached because of the payoff from the insurance company which is listed under other operating income. For this protection the leisure park company has to pay a premium which is included in other operating expenses. That is the reason for the lower profit of $268,700$ Euro in 1991.

5.1.3 Scenario C

This time the company buys the rain option. The differences to scenario A are highlighted in red.

	Scenario C	1991			2002	
		,	%		,	%
	other operating expenses	1.291,4	25,2%		1.286,2	31,6%
	other operating income	419,4	8,2%		944,0	23,2%
M.	EAT	346,6	6,8%	M.	168,7	4,1%

Figure 5.5: P&L cutout for the leisure park in 1991 and 2000, scenario C

As in scenario B this leads to a lower profit in 1991 and to a higher profit in 2000.

Comparing the insurance and the option one observes that there is no significant difference in 2000. The option only leads to a slightly better result. But in 1991 there is a decisive discrepancy. Buying an insurance can provide a profit of $268,700$ Euro; on the other hand the option allows a profit of $346,600$ Euro.
Both hedging tools can protect from impact of disadvantageous years. But the option allows to profit from good years superiorly. The reason for this effect is that the insurance is more expensive compared to the option. This reflects in the other operating expenses and income figures. In 2000 the other operating income is only $95,400$ Euro higher in case of the insurance than in case of the option. But the operating expenses figure is $125,700$ Euro higher. Thus in case of the leisure park the rain option is the hedging tool of choice.

5.2 The dam company

The dam company is a public company as usual in Germany. Therefore it does not have to pay business tax. There are 7 employees. They earn a salary of 40.000 Euro gross at an average. The average revenues amounts 1.1 million Euro per year.
Information from www.finnentrop.de, www.ruhrverband.de and www.ixx.com are taken into account.

The insurance policy bought by the dam company holds for the whole year. It is similar structured as in the case of the leisure park. For every month with less than 300 mm precipitation per m^2 the dam company gets $55,000$ Euro from the insurance company.

In the derivative scenario the dam company buys a put on the same index (summing up the total amount of precipitation) with strike $K = 671.5$ and tick size $s = 2,400$ Euro.

5.2.1 Scenario A

If the dam company does not hedge its rain specific risk the P&L looks as follows:

	Scenario A	1991			2000	
		,	%		,	%
	Revenues	841,4	100,0%		1.107,9	100,0%
A.	Total operating performance	841,4	100,0%	A.	1.107,9	100,0%
B.	Cost of goods sold	157,9	18,8%	B.	167,6	15,1%
C.	Gross margin	683,4	81,2%	C.	940,3	84,9%
D.	Personnel expenses	330,4	39,3%	D.	330,4	29,8%
	other operating expenses	282,7	33,6%		282,7	25,5%
	other operating income	67,7	8,1%		67,7	6,1%
E.	EBITDA	138,0	16,4%	E.	394,9	35,6%
F.	Depreciation and amortisation	138,0	16,4%	F.	140,8	12,7%
G.	EBIT	0,0	0,0%	G.	254,0	22,9%
H.	Financial results	-73,7	-8,8%	H.	-58,7	-5,3%
I.	Income from operations	-73,6	-8,8%	I.	195,4	17,6%
J.	Extraordinary results	0,0	0,0%	J.	0,0	0,0%
K.	Earnings before taxes (EBT)	-73,6	-8,8%	K.	195,4	17,6%
L.	Net income / Net loss	-73,6	-8,8%	L.	146,5	13,2%
M.	EAT	-73,6	-8,8%	M.	146,5	13,2%

Figure 5.6: P&L for the dam in 1991 and 2000, scenario A

One observes that the minimal aim of a public company is not reached in 1991. The company does not work cost-coveringly. This is undesirable although there is a high profit for the year in 2000.

5.2.2 Scenario B

Only the other operating income and expenses figures and the profit for the year are presented in this chapter. Again a more detailed P&L can be found in the appendix B.

	Scenario B	1991			2000	
		,	%		,	%
	other operating expenses	409,3	48,6%		409,3	36,9%
	other operating income	287,7	34,2%		67,7	6,1%
M.	EAT	12,3	1,5%	M.	51,6	4,7%

Figure 5.7: P&L cutout for the dam in 1991 and 2000, scenario B

As in case of the leisure park the insurance can provide more stable profits. Particularly it can preserve from loss in the year 1991.

5.2.3 Scenario C

Hedging the rain specific risk exposure with an option leads to the following changed P&L.

	Scenario C	Gesamtjahr 91			Gesamtjahr 00	
		,	%		,	%
	other operating expenses	368,2	43,7%		368,2	33,4%
	other operating income	248,2	29,5%		67,7	6,1%
M.	EAT	13,3	1,6%	M.	78,3	7,1%

Figure 5.8: P&L cutout for the dam in 1991 and 2000, scenario C

Purchasing the rain option also preserves from loss in 1991. But in comparison to scenario B the profit in 2000 is higher. Thus as in case of the leisure park the option holder can profit from fortunate weather years superiorly compared to the insurance buyer.
Eventually the reason is the same as in the case of the leisure park. The cost performance ratio of the option is better.

Again the rain option proves to be the superior hedging tool.

5.3 Summary

In the previous chapter tools to price rain derivatives were developed and tested. Independent of the validity of the theoretical results these tools only get a practical importance if there is a demand for rain derivatives. In this chapter two companies were presented which depend of rain. It was analysed how rain options influence their P&Ls. One observed that hedging with rain options is advantageous for these companies in contrast to the situation of an open rain specific risk exposure.
Furthermore the option was compared with an insurance against rain. It turned out that in case of these two companies the rain derivative is the hedging tool of choice.

CHAPTER VI

Conclusion

The starting point for this thesis was the question how rain derivatives can be priced. For that 4 different continuous models have been set up. The first model describes a simple mean reversion process:

$$dX_t = \kappa(\theta - X_t)dt + \sigma X_t^p dB_t.$$

In reality there are seasonal trends, that is a property which is not covered by the first model. This problem is resolved by the second model - a mean-reversion model with deterministic mean:

$$dX_t = d\theta(t) + \kappa(\theta(t) - X_t)dt + \sigma X_t^p dB_t.$$

Apart from the seasonality model 1 gives room for improvement. Analysing historic rain data leads to the impression that there are long terms with less respectively more rain than the average. The third model takes this characteristic of rain into account:

$$dX_t = \kappa(\theta - X_t)dt + \sigma X_t^p dB_t^H.$$

Eventually model 4 combines the refinements

$$dX_t = d\theta(t) + \kappa(\theta(t) - X_t)dt + \sigma X_t^p dB_t^H.$$

Note that model 4 covers the three other models.

Afterwards the parameter estimation has been done. It was possible to find sensible calibration techniques for all parameters. As there are no market prices available, the estimators base on historic data.

Different German weather stations have been considered, the developed models fit with every of them. Hereby the adjustment happens with the parameter calibration.

As none of the models is analytically solvable the integration is done numerically. Three schemes have been presented:

- explicit Euler,
- implicit Milstein and
- balanced implicit method.

Hereby the explicit Euler and the balanced implicit method can be applied to ordinary Brownian motion as well as on fractional Brownian motion. The Milstein can only be applied on model 1 and 2 but possesses better convergence properties.

The numerical tests show that the different models lead to different prices. Thus the model refinements make sense. The diffusion parameters influence the prices strongly. Furthermore it turns out that BIM and Milstein differ in their treatment of the volatility parameters.

To show the impact of rain derivatives on a company's performance and to apply the developed models and techniques two case studies are implemented. The influence on the P&L is analysed. Additionally rain options are compared to insurance policies. The options proved to be hedging tool of choice.

This master thesis can only touch on the topic of rain or more generally weather derivatives. So there are many questions and ideas which could not be dealt with. Additionally some new arose.

Firstly the assumption of continuouty is made which allows to model rain quantities. But it is often interesting for a company if it rains or not in a particular period. That is a task which the developed models cannot fulfill. Thus it would be useful to develop discrete models maybe basing on risk theory or insurance mathematics.

The continuous models presented in this thesis can be refined further. So far they are single-factor-models. One idea is to add a second factor like stochastic volatility. A combination of different weather events at different locations in a multi-factor-model could be even more interesting, e. g. to price a swap depending on sun in Munich and rain in Berlin.

Also one could put more effort in the parameter estimation for example by using Kalman filters. This topic is not only theoretically interesting but it is of big importance for praxis as the parameter estimation influences the prices strongly (see chapter 4).

The chapter of numerical tests raises the important question of the choice of the integration scheme. It must be analysed where BIM and Milstein differ and which scheme map the given processes best.

A probably easier question is the question of applicability of the developed models to other weather events. At least in the case of temperatures that should be unproblematic.

APPENDIX A

Ito-calculus

We do not need the whole world of stochastic analysis but some of the results and ideas of Ito-calculus. We want to get a mathematically sound understanding of a stochastic differential equation (SDE)

$$dX_t = a(t, X_t)dt + b(t, X_t)dB_t \tag{A.1}$$

with appropriate functions a and b.
This short introduction or overview bases on [Øks00]. In addition the reader which is interested in stochastic, is referred to [KS88] and [Tod92]. Within this thesis it is assumed that the reader has basic knowledge in probability theory (see for example [Bau92]).

Mathematical preliminaries

Within this subsection the following concepts are repeated:

- Probability space,
- stochastic process,
- filtration,
- adapted,
- martingale.

Definition A.1 Let Ω be a set and $\mathcal{A}$ a family of subsets of Ω. $\mathcal{A}$ is called a σ-algebra on Ω if:

- $\emptyset \in \mathcal{A}$,

- if $A \in \mathcal{A} \Longrightarrow A^C \in \mathcal{A}$,
- if $A_1, A_2, ... \in \mathcal{A} \Longrightarrow \bigcup_{n\geq 1} A_n \in \mathcal{A}$.

If Ω is a given set and $\mathcal{A}$ the associated σ-Algebra then $(\Omega, \mathcal{A})$ is called a measurable space.

Let $\mathcal{G}$ be a family of subsets of Ω then one denotes the σ-algebra generated by $\mathcal{G}$ with

$$\sigma(\mathcal{G}) = \bigcup\{\mathcal{H} : \mathcal{H}\ \sigma\text{-algebra on } \Omega, \mathcal{G} \subset \mathcal{H}\}$$

(the smallest σ-algebra containing $\mathcal{G}$).

Definition A.2 Let $(\Omega, \mathcal{A})$ be a measurable space. A function $P : \mathcal{A} \to [0,1]$ is a probability measure on $(\Omega, \mathcal{A})$ if

- $P(\emptyset) = 0, P(\Omega) = 1$ and
- if $A_1, A_2, ... \in \mathcal{A},$ disjoint $\Longrightarrow P\left(\bigcup_{n\geq 1} A_n\right) = \sum_{n\geq 1} P(A_n)$

Definition A.3

1. One calls $(\Omega, \mathcal{A}, \mathcal{P})$ a probability space if $(\Omega, \mathcal{A})$ is a measurable space and P a probability measure. It is called complete if for all $\hat{A} \in \Omega$ with

$$P^*(\hat{A}) = \inf\{P(A) : \hat{A} \subset A, A \in \mathcal{A}\} = 0$$

holds that $\hat{A} \in \mathcal{A}$.

2. Let $(\Omega, \mathcal{A}, \mathcal{P})$ be a probability space. A function $X : \Omega \to \mathbb{R}^n$ is $\mathcal{A}$-measurable if

$$X^{-1}(U) \in \mathcal{A}$$

for all $U \in \mathbb{R}^n$, $U \in \mathcal{B}$ [1].

3. Let $(\Omega, \mathcal{A}, \mathcal{P})$ be a complete probability space. Then an $\mathcal{A}$-measurable function $X : \Omega \to \mathbb{R}^n$ is called random variable.

Definition A.4 A stochastic process is a parameterised collection of random variables $\{X_t\}_{t\in T}$ defined on $(\Omega, \mathcal{A}, \mathcal{P})$ and taking values in $\mathbb{R}^n$.
(See e. g. [Par72] for more details about stochastic processes.)

[1] $\mathcal{B} = \sigma(\{O \subset \mathbb{R}^n | O \text{ offen }\})$ Borel sets

Sometimes a stochastic processes is written as $X(t, w)$. Hereby one gets a random variable

$$w \longrightarrow X_t(w)$$

for every fixed t. The function

$$t \longrightarrow X(t, w)$$

for fixed w is often called the path of w.
An important example for a stochastic process is the Brownian motion.

Example: A Brownian motion is a function $B(t, \omega) : [0, \infty) \times \Omega \to \mathbb{R}$ fulfilling the following properties:

1. $B(0, w) = 0$ a. s.,

2. the function $B(t, w)$ is continuous for fixed w (continuous paths),

3. for $0 = t_0 \leq t_1 \leq ... \leq t_n < \infty$ the increments of the Brownian motion

$$B(t_1) - B(t_0), ..., B(t_n) - B(t_{n-1})$$

are independently and normally distributed with mean

$$E\left(B(t_{k+1}) - B(t_k)\right) = 0$$

and variance

$$V\left(B(t_{k+1}) - B(t_k)\right) = E\left(\left(B(t_{k+1}) - B(t_k)\right)^2\right) = t_{k+1} - t_k.$$

Definition A.5 Let $B_t(w)$ be a Brownian motion. Then $\mathcal{F}_t$ denotes the smallest σ-algebra containing all sets

$$\{w : B_{t_1}(w) \in \mathcal{F}_1, ..., B_{t_k}(w) \in \mathcal{F}_k\}$$

where $t_j \leq t, F_j \in \mathbb{R}$ Borel sets, $j \leq k = 1, 2, ...$ (We assume that $\mathcal{F}_t$ includes all sets of measure zero.)

Definition A.6 Let $(\mathcal{A}_t)_{t \geq 0}$ be an increasing family of σ-algebras on Ω. A process $f(t, w) : [0, \infty) \times \Omega \to \mathbb{R}^n$ is called $\mathcal{A}_t$-adapted if

$$w \to f(t, w)$$

is $\mathcal{A}_t$-measurable for each $t \geq 0$.

Definition A.7 A family $\mathcal{M} = (\mathcal{M}_t)_{t\geq 0}$ of increasing σ-algebras $\mathcal{M}_t \in \mathcal{A}$, is called a filtration on $(\Omega, \mathcal{F})$.

Definition A.8 A stochastic process $(M_t)_{t\geq 0}$ on $(\Omega, \mathcal{A}, P)$ is a martingale with respect to P and a filtration $(\mathcal{M}_t)_{t\geq 0}$ if

- M_t is $\mathcal{M}_t$-measurable for all t,
- $E[|M_t|] < \infty$ for all t,
- $E[M_s|\mathcal{M}_t] = M_t$ for all $s \geq t$.

Furthermore a semi-martingale $(S_t)_{t\geq 0}$ is a right continuous stochastic process with finite leftside limits which can be presented as

$$S_t = S_0 + A_t + M_t$$

with a process A_t having bounded variation on compact time intervals and a local martingale M_t.

Last we denote $\mathcal{V} = \mathcal{V}(S,T), S \leq T$ as the set of all function

$$f(t,w) : [0,\infty) \times \Omega \to \mathbb{R}$$

such that

- f is $\mathcal{B} \times \mathcal{A}$-measurable, $\mathcal{B}$ Borel σ-algebra on $[0,\infty)$.
- $f(t,w)$ is $\mathcal{A}_t$-adapted.
- $E\left(\int_S^T f(t,w)^2 dt\right) < \infty$.

Ito-integral

The conception of an underlying complete probability space is implicitly assumed in this thesis.

Assuming that the expression (A.1) is sound we can (symbolically) integrate and obtain a stochastic integral equation

$$X_t = X_0 + \int_0^t a(s,X_s)ds + \int_0^t b(s,X_s)dB_s. \tag{A.2}$$

Initial value and deterministic integral are well known concepts. We postulate for a that

$$P\left(\int_0^t |a(s,w)|ds < \infty\right) = 1 \text{ and } a \text{ is } \mathcal{H}_t\text{-adapted.} \tag{A.3}$$

The stochastic integral, an integration after a random variable

$$\int f(t, X_t)dB_t$$

is so far unknown.

Firstly the integral is defined if f is an elementary function. Then this definition is carried forward to general functions.

Let $f \in \mathcal{V}$ given by

$$e(t,w) = \sum_{k\geq 0} \alpha_k(w)\chi_{[t_k,t_{k+1})}(t)$$

be an elementary function. As we want $f \in \mathcal{V}$ it is necessary that α_k is $\mathcal{F}_{t_j}$-measurable. In this case it is natural to define

$$\int_s^t e(t,w)dB_t(w) = \sum_{k\geq 0} \alpha_k(w)(B_{t_{k+1}}(w) - B_{t_k}(w)).$$

Lemma A.9 If $h \in \mathcal{V}$, bounded and with continuous paths a. s. there are elementary functions $h_n \in \mathcal{V}$ such that

$$E\left(\int_s^t (h-h_n)^2 dv\right) \underset{n\longrightarrow\infty}{\longrightarrow} 0$$

Lemma A.10 If $h \in \mathcal{V}$ is bounded then there exist bounded functions $h_n \in \mathcal{V}$ with continuous paths a. s. such that

$$E\left(\int_s^t (h-h_n)^2 dv\right) \underset{n\longrightarrow\infty}{\longrightarrow} 0$$

Lemma A.11 Let $f \in \mathcal{V}$ then there exist bounded functions $f_n \in \mathcal{V}$ such that

$$E\left(\int_s^t (f - f_n)^2 dv\right) \longrightarrow 0 \text{ for } n \longrightarrow \infty$$

Let $s = t_0 \leq t_1 \leq ... \leq t_n = t$ be a partition of $[s,t]$ and $h = \max_{i=1,\dots,n-1} |t_{i+1} - t_i|$. We define for all functions $f \in \mathcal{V}$:

$$\int_s^t f(v,w)dB_v(w) = \lim_{|h|\to 0} \int_s^t f_i(t,w)dB_t(w).$$

This definition is unique, see [Øks00].

Having this in mind one can interpret the SDE (A.1) as the symbolical notation for

$$X_t = X_0 + \int_0^t a(s,X_s)ds + \int_0^t b(s,X_s)dB_s \tag{A.4}$$

with $b \in \mathcal{V}$, a as in (A.3). Such a process is called Ito-process or stochastic integral.

Later on the following calculation rules will be useful:

$$dt \cdot dt = dt \cdot dB_t = dB_t \cdot dt = 0 \text{ and } dB_t \cdot dB_t = dt. \tag{A.5}$$

It easily follows that $d\theta(t) \cdot d\theta(t) = d\theta(t) \cdot dB_t = 0$ for a deterministic function $\theta : \mathbb{R} \to \mathbb{R}$.

One of the most important results in Ito calculus is the Ito theorem (or Ito-formula). It particularly holds for Brownian motion, but in fact it holds for all semi-martingales W_t (see for example [DHPD00]).

Theorem A.12 Let

$$X_t = X_{t_0} + \int_0^t a(s,X_s)ds + \int_0^t b(s,X_s)dW_s$$

be an Ito-process with $X_0 = x_0$. Let $f(t,x) \in \mathcal{C}^{1,2}([0,\infty) \times \mathbb{R})$. Then $Y_t = f(t, X_t)$ is again an Ito-process and it holds

$$df(t, X_t) = \frac{\partial f(t, X_t)}{\partial t} dt + \frac{\partial f(t, X_t)}{\partial x} dX_t + \frac{1}{2} \frac{\partial^2 f(t, X_t)}{\partial x^2} (dX_t)^2.$$

APPENDIX B

Profit and loss accounts

Within the case study in chapter 5 two companies strongly exposed two rain risk have been considered. Three scenarios have been set up. In scenario A there is no hedging. In B the companies hedge with an insurance policy. Lastly the hedging happens with a rain option (scenario C). Two possible rain distributions have been assumed. That is on the one hand the year 1991, and on the other hand 2000.

The different scenarios in the two years are compared by analysing their impact on the profit and loss accounts. The P&L are computed monthly. But in chapter 5 only the totals, sometimes even only cutouts of the totals are presented. In the appendix the interested reader can find the monthly P&L, firstly the leisure park is considered. Afterwards the P&L of the dam company are presented.

	Scenario A	Januar 01		Februar 01		März 01		April 01		Mai 01		Juni 01	
		TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%
	Revenues	2,0	100,0%	2,0	100,0%	2,0	100,0%	279,5	100,0%	624,6	100,0%	657,9	100,0%
A.	Total operating performance	2,0	100,0%	2,0	100,0%	2,0	100,0%	279,5	100,0%	624,6	100,0%	657,9	100,0%
B.	Cost of goods sold	40,3	2015,0%	40,2	2010,0%	45,2	2260,0%	68,0	24,3%	164,9	26,4%	171,6	26,1%
C.	Gross margin	-38,3	-1915,0%	-38,2	-1910,0%	-43,2	-2160,0%	211,6	75,7%	459,7	73,6%	486,3	73,9%
D.	Personnel expenses	54,6	2730,6%	54,6	2730,6%	54,6	2730,6%	88,3	31,6%	88,3	14,1%	95,5	14,5%
	other operating expenses	46,4	2318,6%	46,4	2318,6%	46,4	2318,6%	85,7	30,6%	116,4	18,6%	119,7	18,2%
	other operating income	27,9	1397,1%	27,9	1397,1%	38,1	1904,6%	41,9	15,0%	38,1	6,1%	40,9	6,2%
E.	EBITDA	-111,3	-5567,0%	-111,2	-5562,0%	-106,1	-5304,5%	79,6	28,5%	293,1	46,9%	312,1	47,4%
F.	Depreciation and amortisation	14,0	700,2%	14,0	700,2%	14,0	700,2%	17,4	6,2%	18,0	2,9%	18,0	2,7%
G.	EBIT	-125,3	-6267,2%	-125,2	-6262,2%	-120,1	-6004,7%	62,1	22,2%	275,2	44,1%	294,0	44,7%
H.	Financial results	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-36,0%	-100,3	-16,1%	-100,2	-15,2%
I.	Income from operations	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-38,4	-13,8%	174,9	28,0%	193,8	29,5%
J.	Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K.	Earnings before taxes (EBT)	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-38,4	-13,8%	174,9	28,0%	193,8	29,5%
L.	Net income / Net loss	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-38,4	-13,8%	108,4	17,4%	120,2	18,3%
M.	EAT	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-38,4	-13,8%	108,4	17,4%	120,2	18,3%

	Scenario A	Juli 01		August 01		September 01		Oktober 01		November 01		Dezember 01		1991	
		TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
	Revenues	1.267,7	100,0%	1.332,7	100,0%	448,3	100,0%	499,3	100,0%	2,0	100,0%	2,0	100,0%	5.120,0	100,0%
A.	Total operating performance	1.267,7	100,0%	1.332,7	100,0%	448,3	100,0%	499,3	100,0%	2,0	100,0%	2,0	100,0%	5.120,0	100,0%
B.	Cost of goods sold	293,5	23,2%	306,5	23,0%	84,8	18,9%	89,9	18,0%	40,2	2010,0%	40,2	2010,0%	1.385,4	27,1%
C.	Gross margin	974,2	76,8%	1.026,1	77,0%	363,4	81,1%	409,3	82,0%	-38,2	-1910,0%	-38,2	-1910,0%	3.734,6	72,9%
D.	Personnel expenses	102,8	8,1%	102,8	7,7%	88,3	19,7%	60,5	12,1%	59,4	2972,0%	54,6	2730,6%	904,2	17,7%
	other operating expenses	164,0	12,9%	184,3	13,8%	104,5	23,3%	114,0	22,8%	47,5	2376,1%	46,4	2318,6%	1.121,5	21,9%
	other operating income	41,9	3,3%	41,9	3,1%	27,6	6,2%	37,1	7,4%	27,9	1397,1%	27,9	1397,1%	419,4	8,2%
E.	EBITDA	749,3	59,1%	781,0	58,6%	198,3	44,2%	272,0	54,5%	-117,2	-5860,9%	-111,2	-5562,0%	2.128,3	41,6%
F.	Depreciation and amortisation	19,0	1,5%	19,1	1,4%	17,7	4,0%	17,8	3,6%	14,0	700,2%	14,0	700,2%	197,1	3,9%
G.	EBIT	730,3	57,6%	761,9	57,2%	180,6	40,3%	254,2	50,9%	-131,2	-6561,1%	-125,2	-6262,2%	1.931,1	37,7%
H.	Financial results	-98,7	-7,8%	-98,5	-7,4%	-100,6	-22,4%	-100,6	-20,1%	-100,6	-5028,6%	-100,6	-5028,6%	-1.202,2	-23,5%
I.	Income from operations	631,6	49,8%	663,4	49,8%	80,0	17,9%	153,6	30,8%	-231,8	-11589,6%	-225,8	-11290,7%	728,9	14,2%
J.	Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K.	Earnings before taxes (EBT)	631,6	49,8%	663,4	49,8%	80,0	17,9%	153,6	30,8%	-231,8	-11589,6%	-225,8	-11290,7%	728,9	14,2%
L.	Net income / Net loss	391,6	30,9%	411,3	30,9%	49,6	11,1%	95,2	19,1%	-231,8	-11589,6%	-225,8	-11290,7%	451,9	8,8%
M.	EAT	391,6	30,9%	411,3	30,9%	49,6	11,1%	95,2	19,1%	-231,8	-11589,6%	-225,8	-11290,7%	451,9	8,8%

Figure B.1: P&L leisure park, scenario A, 1991

Scenario A	J		F		M		A		M		J	
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%
Revenues	2,0	100,0%	2,0	100,0%	2,0	100,0%	275,4	100,0%	431,9	100,0%	745,7	100,0%
A. Total operating performance	2,0	100,0%	2,0	100,0%	2,0	100,0%	275,4	100,0%	431,9	100,0%	745,7	100,0%
B. Cost of goods sold	40,3	2015,0%	40,2	2010,0%	45,2	2260,0%	67,5	24,5%	83,2	19,3%	189,1	25,4%
C. Gross margin	-38,3	-1915,0%	-38,2	-1910,0%	-43,2	-2160,0%	207,9	75,5%	348,7	80,7%	556,6	74,6%
D. Personnel expenses	54,6	2730,6%	54,6	2730,6%	54,6	2730,6%	88,3	32,0%	88,3	20,4%	95,5	12,8%
other operating expenses	46,4	2318,6%	46,4	2318,6%	46,4	2318,6%	85,6	31,1%	115,4	26,7%	120,2	16,1%
other operating income	27,9	1397,1%	27,9	1397,1%	38,1	1904,6%	41,9	15,2%	38,1	8,8%	40,9	5,5%
E. EBITDA	-111,3	-5567,0%	-111,2	-5562,0%	-106,1	-5304,5%	75,9	27,6%	183,1	42,4%	381,8	51,2%
F. Depreciation and amortisation	14,0	700,2%	14,0	700,2%	14,0	700,2%	17,4	6,3%	17,7	4,1%	18,2	2,4%
G. EBIT	-125,3	-6267,2%	-125,2	-6262,2%	-120,1	-6004,7%	58,5	21,2%	165,4	38,3%	363,7	48,8%
H. Financial results	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-36,5%	-100,8	-23,3%	-100,0	-13,4%
I. Income from operations	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-42,1	-15,3%	64,7	15,0%	263,7	35,4%
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K. Earnings before taxes (EBT)	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-42,1	-15,3%	64,7	15,0%	263,7	35,4%
L. Net income / Net loss	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-42,1	-15,3%	40,1	9,3%	163,5	21,9%
M. EAT	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-42,1	-15,3%	40,1	9,3%	163,5	21,9%

Scenario A	J		A		S		O		N		D		2000	
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
Revenues	728,5	100,0%	1.154,2	100,0%	330,8	100,0%	398,0	100,0%	2,0	100,0%	2,0	100,0%	4.074,6	100,0%
A. Total operating performance	728,5	100,0%	1.154,2	100,0%	330,8	100,0%	398,0	100,0%	2,0	100,0%	2,0	100,0%	4.074,6	100,0%
B. Cost of goods sold	185,7	25,5%	270,8	23,5%	73,1	22,1%	79,8	20,0%	40,2	2010,0%	40,2	2010,0%	1.155,4	28,4%
C. Gross margin	542,8	74,5%	883,4	76,5%	257,7	77,9%	318,2	80,0%	-38,2	-1910,0%	-38,2	-1910,0%	2.919,2	71,6%
D. Personnel expenses	102,8	14,1%	102,8	8,9%	88,3	26,7%	60,5	15,2%	59,4	2972,0%	54,6	2730,6%	904,2	22,2%
other operating expenses	161,3	22,1%	183,4	15,9%	103,9	31,4%	113,5	28,5%	47,5	2376,1%	46,4	2318,6%	1.116,3	27,4%
other operating income	41,9	5,8%	41,9	3,6%	27,6	8,4%	37,1	9,3%	27,9	1397,1%	27,9	1397,1%	419,4	10,3%
E. EBITDA	320,7	44,0%	639,1	55,4%	93,2	28,2%	181,4	45,6%	-117,2	-5860,9%	-111,2	-5562,0%	1.318,1	32,3%
F. Depreciation and amortisation	18,2	2,5%	18,8	1,6%	17,5	5,3%	17,6	4,4%	14,0	700,2%	14,0	700,2%	195,5	4,8%
G. EBIT	302,5	41,5%	620,2	53,7%	75,7	22,9%	163,7	41,1%	-131,2	-6561,1%	-125,2	-6262,2%	1.122,6	27,6%
H. Financial results	-101,0	-13,9%	-98,9	-8,6%	-100,6	-30,4%	-100,6	-25,3%	-100,6	-5028,6%	-100,6	-5028,6%	-1.205,2	-29,6%
I. Income from operations	201,6	27,7%	521,3	45,2%	-24,9	-7,5%	63,2	15,9%	-231,8	-11589,6%	-225,8	-11290,7%	-82,6	-2,0%
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K. Earnings before taxes (EBT)	201,6	27,7%	521,3	45,2%	-24,9	-7,5%	63,2	15,9%	-231,8	-11589,6%	-225,8	-11290,7%	-82,6	-2,0%
L. Net income / Net loss	125,0	17,2%	323,2	28,0%	-24,9	-7,5%	39,2	9,8%	-231,8	-11589,6%	-225,8	-11290,7%	-82,6	-2,0%
M. EAT	125,0	17,2%	323,2	28,0%	-24,9	-7,5%	39,2	9,8%	-231,8	-11589,6%	-225,8	-11290,7%	-82,6	-2,0%

Figure B.2: P&L leisure park, scenario A, 2000

Scenario B	J		F		M		A		M		J	
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%
Revenues	2,0	100,0%	2,0	100,0%	2,0	100,0%	279,5	100,0%	624,6	100,0%	657,9	100,0%
A. Total operating performance	2,0	100,0%	2,0	100,0%	2,0	100,0%	279,5	100,0%	624,6	100,0%	657,9	100,0%
B. Cost of goods sold	40,3	2015,0%	40,2	2010,0%	45,2	2260,0%	68,0	24,3%	164,9	26,4%	171,6	26,1%
C. Gross margin	-38,3	-1915,0%	-38,2	-1910,0%	-43,2	-2160,0%	211,6	75,7%	459,7	73,6%	486,3	73,9%
D. Personnel expenses	54,6	2730,6%	54,6	2730,6%	54,6	2730,6%	88,3	31,6%	88,3	14,1%	95,5	14,5%
other operating expenses	46,4	2318,6%	46,4	2318,6%	46,4	2318,6%	127,9	45,7%	158,6	25,4%	161,9	24,6%
other operating income	27,9	1397,1%	27,9	1397,1%	38,1	1904,6%	41,9	15,0%	38,1	6,1%	40,9	6,2%
E. EBITDA	-111,3	-5567,0%	-111,2	-5562,0%	-106,1	-5304,5%	37,4	13,4%	250,9	40,2%	269,9	41,0%
F. Depreciation and amortisation	14,0	700,2%	14,0	700,2%	14,0	700,2%	17,4	6,2%	18,0	2,9%	18,0	2,7%
G. EBIT	-125,3	-6267,2%	-125,2	-6262,2%	-120,1	-6004,7%	19,9	7,1%	232,9	37,3%	251,8	38,3%
H. Financial results	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-36,0%	-100,3	-16,1%	-100,2	-15,2%
I. Income from operations	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-80,7	-28,9%	132,7	21,2%	151,6	23,0%
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K. Earnings before taxes (EBT)	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-80,7	-28,9%	132,7	21,2%	151,6	23,0%
L. Net income / Net loss	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-80,7	-28,9%	82,3	13,2%	94,0	14,3%
M. EAT	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-80,7	-28,9%	82,3	13,2%	94,0	14,3%

Scenario B	J		A		S		O		N		D		2000	
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
Revenues	1.267,7	100,0%	1.332,7	100,0%	448,3	100,0%	499,3	100,0%	2,0	100,0%	2,0	100,0%	5.120,0	100,0%
A. Total operating performance	1.267,7	100,0%	1.332,7	100,0%	448,3	100,0%	499,3	100,0%	2,0	100,0%	2,0	100,0%	5.120,0	100,0%
B. Cost of goods sold	293,5	23,2%	306,5	23,0%	84,8	18,9%	89,9	18,0%	40,2	2010,0%	40,2	2010,0%	1.385,4	27,1%
C. Gross margin	974,2	76,8%	1.026,1	77,0%	363,4	81,1%	409,3	82,0%	-38,2	-1910,0%	-38,2	-1910,0%	3.734,6	72,9%
D. Personnel expenses	102,8	8,1%	102,8	7,7%	88,3	19,7%	60,5	12,1%	59,4	2972,0%	54,6	2730,6%	904,2	17,7%
other operating expenses	206,2	16,3%	226,5	17,0%	146,7	32,7%	156,3	31,3%	47,5	2376,1%	46,4	2318,6%	1.417,1	27,7%
other operating income	41,9	3,3%	41,9	3,1%	27,6	6,2%	37,1	7,4%	27,9	1397,1%	27,9	1397,1%	419,4	8,2%
E. EBITDA	707,1	55,8%	738,8	55,4%	156,1	34,8%	229,7	46,0%	-117,2	-5860,9%	-111,2	-5562,0%	1.832,7	35,8%
F. Depreciation and amortisation	19,0	1,5%	19,1	1,4%	17,7	4,0%	17,8	3,6%	14,0	700,2%	14,0	700,2%	197,1	3,9%
G. EBIT	688,1	54,3%	719,6	54,0%	138,4	30,9%	211,9	42,5%	-131,2	-6561,1%	-125,2	-6262,2%	1.635,6	31,9%
H. Financial results	-98,7	-7,8%	-98,5	-7,4%	-100,6	-22,4%	-100,6	-20,1%	-100,6	-5028,6%	-100,6	-5028,6%	-1.202,2	-23,5%
I. Income from operations	589,4	46,5%	621,1	46,6%	37,8	8,4%	111,4	22,3%	-231,8	-11589,6%	-225,8	-11290,7%	433,4	8,5%
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K. Earnings before taxes (EBT)	589,4	46,5%	621,1	46,6%	37,8	8,4%	111,4	22,3%	-231,8	-11589,6%	-225,8	-11290,7%	433,4	8,5%
L. Net income / Net loss	365,4	28,8%	385,1	28,9%	23,5	5,2%	69,1	13,8%	-231,8	-11589,6%	-225,8	-11290,7%	268,7	5,2%
M. EAT	365,4	28,8%	385,1	28,9%	23,5	5,2%	69,1	13,8%	-231,8	-11589,6%	-225,8	-11290,7%	268,7	5,2%

Figure B.3: P& L leisure park, scenario B, 1991

	Scenario B	J		F		M		A		M		J	
		TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%
	Umsatzerlöse	2,0	100,0%	2,0	100,0%	2,0	100,0%	275,4	100,0%	431,9	100,0%	745,7	100,0%
A.	Gesamtleistung	2,0	100,0%	2,0	100,0%	2,0	100,0%	275,4	100,0%	431,9	100,0%	745,7	100,0%
B.	Materialaufwand	40,3	2015,0%	40,2	2010,0%	45,2	2260,0%	67,5	24,5%	83,2	19,3%	189,1	25,4%
C.	Rohertrag	-38,3	-1915,0%	-38,2	-1910,0%	-43,2	-2160,0%	207,9	75,5%	348,7	80,7%	556,6	74,6%
D.	Personalaufwand	54,6	2730,6%	54,6	2730,6%	54,6	2730,6%	88,3	32,0%	88,3	20,4%	95,5	12,8%
	sonstige betriebliche Aufwendungen	46,4	2318,6%	46,4	2318,6%	46,4	2318,6%	127,9	46,4%	157,6	36,5%	162,4	21,8%
	sonstige betriebliche Erträge	27,9	1397,1%	27,9	1397,1%	38,1	1904,6%	351,9	127,8%	38,1	8,8%	40,9	5,5%
E.	EBITDA	-111,3	-5567,0%	-111,2	-5562,0%	-106,1	-5304,5%	343,7	124,8%	140,9	32,6%	339,6	45,5%
F.	Abschreibungen	14,0	700,2%	14,0	700,2%	14,0	700,2%	17,4	6,3%	17,7	4,1%	18,2	2,4%
G.	EBIT	-125,3	-6267,2%	-125,2	-6262,2%	-120,1	-6004,7%	326,2	118,4%	123,2	28,5%	321,4	43,1%
H.	Finanzergebnis	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-36,5%	-100,8	-23,3%	-100,0	-13,4%
I.	Ergebnis der gewöhnlichen Geschäftstätigkeit	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	225,7	81,9%	22,4	5,2%	221,5	29,7%
J.	Außerordentliches Ergebnis	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K.	Ergebnis vor Steuern (EBT)	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	225,7	81,9%	22,4	5,2%	221,5	29,7%
L.	Jahresüberschuss / -fehlbetrag (EAT)	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	139,9	50,8%	13,9	3,2%	137,3	18,4%
M.	Jahresüberschuss/-fehlbetrag (EAT) (nach Gewinnanteil fremder Gesellschafter)	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	139,9	50,8%	13,9	3,2%	137,3	18,4%

	Scenario B	J		A		S		O		N		D		Gesamtjahr 1991	
		TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
	Umsatzerlöse	728,5	100,0%	1.154,2	100,0%	330,8	100,0%	398,0	100,0%	2,0	100,0%	2,0	100,0%	4.074,6	100,0%
A.	Gesamtleistung	728,5	100,0%	1.154,2	100,0%	330,8	100,0%	398,0	100,0%	2,0	100,0%	2,0	100,0%	4.074,6	100,0%
B.	Materialaufwand	185,7	25,5%	270,8	23,5%	73,1	22,1%	79,8	20,0%	40,2	2010,0%	40,2	2010,0%	1.155,4	28,4%
C.	Rohertrag	542,8	74,5%	883,4	76,5%	257,7	77,9%	318,2	80,0%	-38,2	-1910,0%	-38,2	-1910,0%	2.919,2	71,6%
D.	Personalaufwand	102,8	14,1%	102,8	8,9%	88,3	26,7%	60,5	15,2%	59,4	2972,0%	54,6	2730,6%	904,2	22,2%
	sonstige betriebliche Aufwendungen	203,5	27,9%	225,6	19,5%	146,1	44,2%	155,7	39,1%	47,5	2376,1%	46,4	2318,6%	1.411,9	34,7%
	sonstige betriebliche Erträge	351,9	48,3%	41,9	3,6%	27,6	8,4%	37,1	9,3%	27,9	1397,1%	27,9	1397,1%	1.039,4	25,5%
E.	EBITDA	588,5	80,8%	596,9	51,7%	51,0	15,4%	139,1	35,0%	-117,2	-5860,9%	-111,2	-5562,0%	1.642,5	40,3%
F.	Abschreibungen	18,2	2,5%	18,8	1,6%	17,5	5,3%	17,6	4,4%	14,0	700,2%	14,0	700,2%	195,5	4,8%
G.	EBIT	570,3	78,3%	578,0	50,1%	33,5	10,1%	121,5	30,5%	-131,2	-6561,1%	-125,2	-6262,2%	1.447,0	35,5%
H.	Finanzergebnis	-101,0	-13,9%	-98,9	-8,6%	-100,6	-30,4%	-100,6	-25,3%	-100,6	-5028,6%	-100,6	-5028,6%	-1.205,2	-29,6%
I.	Ergebnis der gewöhnlichen Geschäftstätigkeit	469,3	64,4%	479,1	41,5%	-67,1	-20,3%	20,9	5,3%	-231,8	-11589,6%	-225,8	-11290,7%	241,8	5,9%
J.	Außerordentliches Ergebnis	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K.	Ergebnis vor Steuern (EBT)	469,3	64,4%	479,1	41,5%	-67,1	-20,3%	20,9	5,3%	-231,8	-11589,6%	-225,8	-11290,7%	241,8	5,9%
L.	Jahresüberschuss / -fehlbetrag (EAT)	291,0	39,9%	297,0	25,7%	-67,1	-20,3%	13,0	3,3%	-231,8	-11589,6%	-225,8	-11290,7%	149,9	3,7%
M.	Jahresüberschuss/-fehlbetrag (EAT) (nach Gewinnanteil fremder Gesellschafter)	291,0	39,9%	297,0	25,7%	-67,1	-20,3%	13,0	3,3%	-231,8	-11589,6%	-225,8	-11290,7%	149,9	3,7%

Figure B.4: P& L leisure park, scenario B, 2000

Scenario C	J		F		M		A		M		J		J		A		S		O		N		D		199
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,
Revenues	2,0	100,0%	2,0	100,0%	2,0	100,0%	279,5	100,0%	624,6	100,0%	657,9	100,0%	1.267,7	100,0%	1.332,7	100,0%	448,3	100,0%	499,3	100,0%	2,0	100,0%	2,0	100,0%	5.120,0
A. Total operating performance	2,0	100,0%	2,0	100,0%	2,0	100,0%	279,5	100,0%	624,6	100,0%	657,9	100,0%	1.267,7	100,0%	1.332,7	100,0%	448,3	100,0%	499,3	100,0%	2,0	100,0%	2,0	100,0%	5.120,0
B. Cost of goods sold	40,3	2015,0%	40,2	2010,0%	45,2	2260,0%	68,0	24,3%	164,9	26,4%	171,6	26,1%	293,5	23,2%	306,5	23,0%	84,8	18,9%	89,9	18,0%	40,2	2010,0%	40,2	2010,0%	1.385,4
C. Gross margin	-38,3	-1915,0%	-38,2	-1910,0%	-43,2	-2160,0%	211,6	75,7%	459,7	73,6%	486,3	73,9%	974,2	76,8%	1.026,1	77,0%	363,4	81,1%	409,3	82,0%	-38,2	-1910,0%	-38,2	-1910,0%	3.734,6
D. Personnel expenses	54,6	2730,6%	54,6	2730,6%	54,6	2730,6%	88,3	31,6%	88,3	14,1%	95,5	14,5%	102,8	8,1%	102,8	7,7%	88,3	19,7%	60,5	12,1%	59,4	2972,0%	54,6	2730,6%	904,2
other operating expenses	46,4	2318,6%	46,4	2318,6%	46,4	2318,6%	109,9	39,3%	140,6	22,5%	144,0	21,9%	188,2	14,8%	208,5	15,6%	128,8	28,7%	138,3	27,7%	47,5	2376,1%	46,4	2318,6%	1.291,4
other operating income	27,9	1397,1%	27,9	1397,1%	38,1	1904,6%	41,9	15,0%	38,1	6,1%	40,9	6,2%	41,9	3,3%	41,9	3,1%	27,6	6,2%	37,1	7,4%	27,9	1397,1%	27,9	1397,1%	419,4
E. EBITDA	-111,3	-5567,0%	-111,2	-5562,0%	-106,1	-5304,5%	55,3	19,8%	268,9	43,0%	287,8	43,7%	725,1	57,2%	756,7	56,8%	174,1	38,8%	247,7	49,6%	-117,2	-5860,9%	-111,2	-5562,0%	1.958,4
F. Depreciation and amortisation	14,0	700,2%	14,0	700,2%	14,0	700,2%	17,4	6,2%	18,0	2,9%	18,0	2,7%	19,0	1,5%	19,1	1,4%	17,7	4,0%	17,8	3,6%	14,0	700,2%	14,0	700,2%	197,1
G. EBIT	-125,3	-6267,2%	-125,2	-6262,2%	-120,1	-6004,7%	37,9	13,5%	250,9	40,2%	269,8	41,0%	706,0	55,7%	737,6	55,3%	156,4	34,9%	229,9	46,0%	-131,2	-6561,1%	-125,2	-6262,2%	1.761,3
H. Financial results	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-36,0%	-100,3	-16,1%	-100,2	-15,2%	-98,7	-7,8%	-98,5	-7,4%	-100,6	-22,4%	-100,6	-20,1%	-100,6	-5028,6%	-100,6	-5028,6%	-1.202,2
I. Income from operations	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-62,7	-22,4%	150,6	24,1%	169,6	25,8%	607,4	47,9%	639,1	48,0%	55,8	12,4%	129,3	25,9%	-231,8	-11589,6%	-225,8	-11290,7%	559,1
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0
K. Earnings before taxes (EBT)	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-62,7	-22,4%	150,6	24,1%	169,6	25,8%	607,4	47,9%	639,1	48,0%	55,8	12,4%	129,3	25,9%	-231,8	-11589,6%	-225,8	-11290,7%	559,1
L. Net income / Net loss	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-62,7	-22,4%	93,4	15,0%	105,1	16,0%	376,6	29,7%	396,2	29,7%	34,6	7,7%	80,2	16,1%	-231,8	-11589,6%	-225,8	-11290,7%	346,6
M. EAT	-225,9	-11295,7%	-225,8	-11290,7%	-220,7	-11033,2%	-62,7	-22,4%	93,4	15,0%	105,1	16,0%	376,6	29,7%	396,2	29,7%	34,6	7,7%	80,2	16,1%	-231,8	-11589,6%	-225,8	-11290,7%	346,6

Figure B.5: P& L leisure park, scenario C, 1991

Scenario C	J		F		M		A		M		J	
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%
Revenues	2,0	100,0%	2,0	100,0%	2,0	100,0%	275,4	100,0%	431,9	100,0%	745,7	100,0%
A. Total operating performance	2,0	100,0%	2,0	100,0%	2,0	100,0%	275,4	100,0%	431,9	100,0%	745,7	100,0%
B. Cost of goods sold	40,3	2015,0%	40,2	2010,0%	45,2	2260,0%	67,5	24,5%	83,2	19,3%	189,1	25,4%
C. Gross margin	-38,3	-1915,0%	-38,2	-1910,0%	-43,2	-2160,0%	207,9	75,5%	348,7	80,7%	556,6	74,6%
D. Personnel expenses	54,6	2730,6%	54,6	2730,6%	54,6	2730,6%	88,3	32,0%	88,3	20,4%	95,5	12,8%
other operating expenses	46,4	2318,6%	46,4	2318,6%	46,4	2318,6%	109,9	39,9%	139,7	32,3%	144,4	19,4%
other operating income	27,9	1397,1%	27,9	1397,1%	98,2	4912,1%	102,0	37,0%	98,2	22,7%	101,1	13,6%
E. EBITDA	-111,3	-5567,0%	-111,2	-5562,0%	-45,9	-2297,1%	111,8	40,6%	219,0	50,7%	417,7	56,0%
F. Depreciation and amortisation	14,0	700,2%	14,0	700,2%	14,0	700,2%	17,4	6,3%	17,7	4,1%	18,2	2,4%
G. EBIT	-125,3	-6267,2%	-125,2	-6262,2%	-59,9	-2997,2%	94,3	34,3%	201,3	46,6%	399,5	53,6%
H. Financial results	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-5028,6%	-100,6	-36,5%	-100,8	-23,3%	-100,0	-13,4%
I. Income from operations	-225,9	-11295,7%	-225,8	-11290,7%	-160,5	-8025,8%	-6,2	-2,3%	100,5	23,3%	299,6	40,2%
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K. Earnings before taxes (EBT)	-225,9	-11295,7%	-225,8	-11290,7%	-160,5	-8025,8%	-6,2	-2,3%	100,5	23,3%	299,6	40,2%
L. Net income / Net loss	-225,9	-11295,7%	-225,8	-11290,7%	-160,5	-8025,8%	-6,2	-2,3%	62,3	14,4%	185,7	24,9%
M. EAT	-225,9	-11295,7%	-225,8	-11290,7%	-160,5	-8025,8%	-6,2	-2,3%	62,3	14,4%	185,7	24,9%

Scenario C	J		A		S		O		N		D		2000	
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
Revenues	728,5	100,0%	1.154,2	100,0%	352,1	100,0%	398,0	100,0%	2,0	100,0%	2,0	100,0%	4.095,8	100,0%
A. Total operating performance	728,5	100,0%	1.154,2	100,0%	352,1	100,0%	398,0	100,0%	2,0	100,0%	2,0	100,0%	4.095,8	100,0%
B. Cost of goods sold	185,7	25,5%	270,8	23,5%	75,2	21,4%	79,8	20,0%	40,2	2010,0%	40,2	2010,0%	1.157,5	28,3%
C. Gross margin	542,8	74,5%	883,4	76,5%	276,8	78,6%	318,2	80,0%	-38,2	-1910,0%	-38,2	-1910,0%	2.938,3	71,7%
D. Personnel expenses	102,8	14,1%	102,8	8,9%	88,3	25,1%	60,5	15,2%	59,4	2972,0%	54,6	2730,6%	904,2	22,1%
other operating expenses	185,5	25,5%	207,7	18,0%	128,3	36,4%	137,8	34,6%	47,5	2376,1%	46,4	2318,6%	1.286,3	31,4%
other operating income	102,0	14,0%	102,0	8,8%	87,8	24,9%	97,3	24,4%	27,9	1397,1%	27,9	1397,1%	900,6	22,0%
E. EBITDA	356,6	48,9%	675,0	58,5%	148,1	42,1%	217,2	54,6%	-117,2	-5860,9%	-111,2	-5562,0%	1.648,4	40,2%
F. Depreciation and amortisation	18,2	2,5%	18,8	1,6%	17,6	5,0%	17,6	4,4%	14,0	700,2%	14,0	700,2%	195,5	4,8%
G. EBIT	338,4	46,5%	656,1	56,8%	130,5	37,1%	199,6	50,1%	-131,2	-6561,1%	-125,2	-6262,2%	1.452,9	35,5%
H. Financial results	-101,0	-13,9%	-98,9	-8,6%	-100,6	-28,6%	-100,6	-25,3%	-100,6	-5028,6%	-100,6	-5028,6%	-1.205,2	-29,4%
I. Income from operations	237,4	32,6%	557,2	48,3%	30,0	8,5%	99,0	24,9%	-231,8	-11589,6%	-225,8	-11290,7%	247,7	6,0%
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K. Earnings before taxes (EBT)	237,4	32,6%	557,2	48,3%	30,0	8,5%	99,0	24,9%	-231,8	-11589,6%	-225,8	-11290,7%	247,7	6,0%
L. Net income / Net loss	147,2	20,2%	345,5	29,9%	18,6	5,3%	61,4	15,4%	-231,8	-11589,6%	-225,8	-11290,7%	153,6	3,7%
M. EAT	147,2	20,2%	345,5	29,9%	18,6	5,3%	61,4	15,4%	-231,8	-11589,6%	-225,8	-11290,7%	153,6	3,7%

Figure B.6: P& L leisure park, scenario C, 2000

	Scenario A	J		F		M		A		M		J		J		A		S		O		N		D		1991	
		TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
	Revenues	37,5	100,0%	21,1	100,0%	23,6	100,0%	100,6	100,0%	92,7	100,0%	129,8	100,0%	61,7	100,0%	60,4	100,0%	62,6	100,0%	93,8	100,0%	86,7	100,0%	71,1	100,0%	841,4	100,0%
A.	Total operating performance	37,5	100,0%	21,1	100,0%	23,6	100,0%	100,6	100,0%	92,7	100,0%	129,8	100,0%	61,7	100,0%	60,4	100,0%	62,6	100,0%	93,8	100,0%	86,7	100,0%	71,1	100,0%	841,4	100,0%
B.	Cost of goods sold	10,9	29,2%	10,8	51,1%	10,8	45,8%	14,6	14,5%	14,3	15,4%	15,8	12,1%	13,0	21,1%	13,0	21,5%	13,1	20,9%	14,3	15,3%	14,0	16,2%	13,4	18,9%	157,9	18,8%
C.	Gross margin	26,6	70,8%	10,3	48,9%	12,8	54,2%	86,0	85,5%	78,4	84,6%	114,1	87,9%	48,6	78,9%	47,4	78,5%	49,5	79,1%	79,4	84,7%	72,7	83,8%	57,7	81,1%	683,4	81,2%
D.	Personnel expenses	27,5	73,4%	27,5	130,7%	27,5	116,8%	27,5	27,4%	27,5	29,7%	27,5	21,2%	27,5	44,6%	27,5	45,6%	27,5	44,0%	27,5	29,4%	27,5	31,7%	27,5	38,7%	330,4	39,3%
	other operating expenses	23,9	63,8%	16,0	76,1%	15,2	64,6%	25,2	25,0%	25,4	27,4%	24,1	18,6%	21,1	34,3%	23,4	38,8%	25,6	40,9%	33,1	35,3%	24,8	28,5%	24,9	35,1%	282,7	33,6%
	other operating income	6,6	17,6%	3,9	18,4%	4,1	17,2%	4,0	4,0%	4,2	4,5%	4,9	3,8%	5,9	9,5%	6,2	10,2%	6,3	10,1%	6,7	7,2%	7,4	8,5%	7,6	10,7%	67,7	8,1%
E.	EBITDA	-18,3	-48,7%	-29,4	-139,5%	-25,9	-110,0%	37,3	37,1%	29,7	32,0%	67,3	51,8%	5,9	9,5%	2,6	4,3%	2,7	4,3%	25,6	27,3%	27,8	32,1%	12,8	18,0%	138,0	16,4%
F.	Depreciation and amortisation	9,4	25,1%	9,2	43,8%	9,3	39,3%	13,1	13,0%	13,0	14,0%	13,4	10,3%	12,7	20,5%	12,6	20,9%	12,7	20,3%	13,0	13,9%	9,9	11,4%	9,8	13,7%	138,0	16,4%
G.	EBIT	-27,7	-73,8%	-38,6	-183,3%	-35,2	-149,2%	24,2	24,1%	16,7	18,0%	53,9	41,5%	-6,8	-11,0%	-10,1	-16,7%	-10,0	-16,0%	12,6	13,4%	17,9	20,6%	3,0	4,3%	0,0	0,0%
H.	Financial results	-7,6	-20,2%	-4,9	-23,1%	-4,7	-19,9%	-4,6	-4,5%	-6,2	-6,7%	-4,5	-3,4%	-5,4	-8,8%	-6,6	-10,9%	-8,2	-13,1%	-11,8	-12,6%	-4,6	-5,3%	-4,7	-6,6%	-73,7	-8,8%
I.	Income from operations	-35,2	-94,0%	-43,5	-206,4%	-39,9	-169,1%	19,6	19,5%	10,5	11,3%	49,5	38,1%	-12,2	-19,8%	-16,6	-27,5%	-18,2	-29,1%	0,8	0,8%	13,3	15,3%	-1,6	-2,3%	-73,6	-8,8%
J.	Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K.	Earnings before taxes (EBT)	-35,2	-94,0%	-43,5	-206,4%	-39,9	-169,1%	19,6	19,5%	10,5	11,3%	49,5	38,1%	-12,2	-19,8%	-16,6	-27,5%	-18,2	-29,1%	0,8	0,8%	13,3	15,3%	-1,6	-2,3%	-73,6	-8,8%
L.	Net income / Net loss	-35,2	-94,0%	-43,5	-206,4%	-39,9	-169,1%	14,7	14,6%	7,9	8,5%	37,1	28,6%	-12,2	-19,8%	-16,6	-27,5%	-18,2	-29,1%	0,6	0,6%	10,0	11,5%	-1,6	-2,3%	-73,6	-8,8%
M.	EAT	-35,2	-94,0%	-43,5	-206,4%	-39,9	-169,1%	14,7	14,6%	7,9	8,5%	37,1	28,6%	-12,2	-19,8%	-16,6	-27,5%	-18,2	-29,1%	0,6	0,6%	10,0	11,5%	-1,6	-2,3%	-73,6	-8,8%

Figure B.7: P& L dam, scenario A, 1991

	Scenario A	J		F		M		A		M		J	
		TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%
	Revenues	57,8	100,0%	42,6	100,0%	28,5	100,0%	104,1	100,0%	172,1	100,0%	110,7	100,0%
A.	Total operating performance	57,8	100,0%	42,6	100,0%	28,5	100,0%	104,1	100,0%	172,1	100,0%	110,7	100,0%
B.	Cost of goods sold	12,9	22,3%	11,0	25,8%	10,8	38,0%	14,7	14,2%	17,4	10,1%	15,0	13,5%
C.	Gross margin	44,9	77,7%	31,6	74,2%	17,7	62,0%	89,3	85,8%	154,6	89,9%	95,7	86,5%
D.	Personnel expenses	27,5	47,7%	27,5	64,6%	27,5	96,5%	27,5	26,5%	27,5	16,0%	27,5	24,9%
	other operating expenses	23,9	41,4%	16,0	37,6%	15,2	53,3%	25,2	24,2%	25,4	14,8%	24,1	21,8%
	other operating income	6,6	11,5%	3,9	9,1%	4,1	14,2%	4,0	3,9%	4,2	2,4%	4,9	4,4%
E.	EBITDA	0,0	0,1%	-8,1	-18,9%	-21,0	-73,6%	40,6	39,0%	105,9	61,5%	49,0	44,2%
F.	Depreciation and amortisation	9,6	16,7%	9,5	22,2%	9,3	32,6%	13,1	12,6%	13,8	8,0%	13,2	11,9%
G.	EBIT	-9,6	-16,6%	-17,5	-41,1%	-30,3	-106,1%	27,5	26,5%	92,1	53,5%	35,8	32,3%
H.	Financial results	-5,0	-8,6%	-4,7	-11,0%	-4,7	-16,4%	-4,6	-4,4%	-4,3	-2,5%	-4,5	-4,1%
I.	Income from operations	-14,5	-25,2%	-22,2	-52,1%	-35,0	-122,6%	23,0	22,1%	87,7	51,0%	31,2	28,2%
J.	Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K.	Earnings before taxes (EBT)	-14,5	-25,2%	-22,2	-52,1%	-35,0	-122,6%	23,0	22,1%	87,7	51,0%	31,2	28,2%
L.	Net income / Net loss	-14,5	-25,2%	-22,2	-52,1%	-35,0	-122,6%	17,2	16,6%	65,8	38,2%	23,4	21,2%
M.	EAT	-14,5	-25,2%	-22,2	-52,1%	-35,0	-122,6%	17,2	16,6%	65,8	38,2%	23,4	21,2%

	Scenario A	J		A		S		O		N		D		1991	
		TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
	Revenues	122,6	100,0%	82,8	100,0%	113,5	100,0%	173,3	100,0%	56,9	100,0%	43,2	100,0%	1.107,9	100,0%
A.	Total operating performance	122,6	100,0%	82,8	100,0%	113,5	100,0%	173,3	100,0%	56,9	100,0%	43,2	100,0%	1.107,9	100,0%
B.	Cost of goods sold	15,5	12,6%	13,9	16,8%	15,1	13,3%	17,5	10,1%	12,8	22,6%	11,0	25,5%	167,6	15,1%
C.	Gross margin	107,1	87,4%	68,9	83,2%	98,4	86,7%	155,8	89,9%	44,0	77,4%	32,2	74,5%	940,3	84,9%
D.	Personnel expenses	27,5	22,5%	27,5	33,3%	27,5	24,3%	27,5	15,9%	27,5	48,4%	27,5	63,7%	330,4	29,8%
	other operating expenses	21,2	17,3%	23,4	28,3%	25,6	22,6%	33,1	19,1%	24,8	43,5%	24,9	57,7%	282,7	25,5%
	other operating income	5,9	4,8%	6,2	7,4%	6,3	5,6%	6,7	3,9%	7,4	13,0%	7,6	17,5%	67,7	6,1%
E.	EBITDA	64,3	52,5%	24,1	29,1%	51,6	45,4%	101,9	58,8%	-0,8	-1,5%	-12,7	-29,3%	394,9	35,6%
F.	Depreciation and amortisation	13,3	10,9%	12,9	15,6%	13,2	11,6%	13,8	8,0%	9,6	16,9%	9,5	21,9%	140,8	12,7%
G.	EBIT	51,0	41,6%	11,2	13,5%	38,4	33,8%	88,0	50,8%	-10,4	-18,4%	-22,1	-51,2%	254,0	22,9%
H.	Financial results	-4,5	-3,7%	-4,6	-5,6%	-4,5	-4,0%	-4,3	-2,5%	-6,1	-10,6%	-6,8	-15,8%	-58,7	-5,3%
I.	Income from operations	46,5	37,9%	6,6	8,0%	33,8	29,8%	83,7	48,3%	-16,5	-29,0%	-29,0	-67,1%	195,4	17,6%
J.	Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K.	Earnings before taxes (EBT)	46,5	37,9%	6,6	8,0%	33,8	29,8%	83,7	48,3%	-16,5	-29,0%	-29,0	-67,1%	195,4	17,6%
L.	Net income / Net loss	34,9	28,5%	4,9	6,0%	25,4	22,4%	62,8	36,2%	-16,5	-29,0%	-29,0	-67,1%	146,5	13,2%
M.	EAT	34,9	28,5%	4,9	6,0%	25,4	22,4%	62,8	36,2%	-16,5	-29,0%	-29,0	-67,1%	146,5	13,2%

Figure B.8: P& L dam, scenario A, 2000

	Scenario B	J		F		M		A		M		J		J		A		S		O		N		D		1991	
		TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
	Revenues	37,5	100,0%	21,1	100,0%	23,6	100,0%	100,6	100,0%	92,7	100,0%	129,8	100,0%	61,7	100,0%	60,4	100,0%	62,6	100,0%	93,8	100,0%	86,7	100,0%	71,1	100,0%	841,4	100,0%
A.	Total operating performance	37,5	100,0%	21,1	100,0%	23,6	100,0%	100,6	100,0%	92,7	100,0%	129,8	100,0%	61,7	100,0%	60,4	100,0%	62,6	100,0%	93,8	100,0%	86,7	100,0%	71,1	100,0%	841,4	100,0%
B.	Cost of goods sold	10,9	29,2%	10,8	51,1%	10,8	45,8%	14,6	14,5%	14,3	15,4%	15,8	12,1%	13,0	21,1%	13,0	21,5%	13,1	20,9%	14,3	15,3%	14,0	16,2%	13,4	18,9%	157,9	18,8%
C.	Gross margin	26,6	70,8%	10,3	48,9%	12,8	54,2%	86,0	85,5%	78,4	84,6%	114,1	87,9%	48,6	78,9%	47,4	78,5%	49,5	79,1%	79,4	84,7%	72,7	83,8%	57,7	81,1%	683,4	81,2%
D.	Personnel expenses	27,5	73,4%	27,5	130,7%	27,5	116,8%	27,5	27,4%	27,5	29,7%	27,5	21,2%	27,5	44,6%	27,5	45,6%	27,5	44,0%	27,5	29,4%	27,5	31,7%	27,5	38,7%	330,4	39,3%
	other operating expenses	34,5	91,9%	26,6	126,1%	25,8	109,3%	35,7	35,5%	35,9	38,8%	34,6	26,7%	31,7	51,4%	34,0	56,3%	36,2	57,8%	43,6	46,5%	35,3	40,7%	35,5	49,9%	409,3	48,6%
	other operating income	61,6	164,3%	58,9	279,5%	4,1	17,2%	4,0	4,0%	59,2	63,9%	4,9	3,8%	5,9	9,5%	6,2	10,2%	6,3	10,1%	61,7	65,9%	7,4	8,5%	7,6	10,7%	287,7	34,2%
E.	EBITDA	26,2	69,8%	15,1	71,5%	-36,5	-154,7%	26,7	26,6%	74,1	80,0%	56,8	43,7%	-4,7	-7,6%	-8,0	-13,2%	-7,9	-12,6%	70,0	74,7%	17,3	19,9%	2,3	3,2%	231,5	27,5%
F.	Depreciation and amortisation	9,4	25,1%	9,2	43,8%	9,3	39,3%	13,1	13,0%	13,0	14,0%	13,4	10,3%	12,7	20,5%	12,6	20,9%	12,7	20,3%	13,0	13,9%	9,9	11,4%	9,8	13,7%	138,0	16,4%
G.	EBIT	16,8	44,8%	5,8	27,7%	-45,7	-194,0%	13,7	13,6%	61,1	66,0%	43,4	33,4%	-17,3	-28,1%	-20,6	-34,1%	-20,5	-32,8%	57,0	60,8%	7,4	8,5%	-7,5	-10,5%	93,5	11,1%
H.	Financial results	-7,6	-20,2%	-4,9	-23,1%	-4,7	-19,9%	-4,6	-4,5%	-6,2	-6,7%	-4,5	-3,4%	-5,4	-8,8%	-6,6	-10,9%	-8,2	-13,1%	-11,8	-12,6%	-4,6	-5,3%	-4,7	-6,6%	-73,7	-8,8%
I.	Income from operations	9,2	24,6%	1,0	4,6%	-50,4	-213,9%	9,1	9,0%	55,0	59,3%	38,9	30,0%	-22,8	-36,9%	-27,2	-45,0%	-28,8	-46,0%	45,2	48,2%	2,7	3,2%	-12,2	-17,1%	19,8	2,4%
J.	Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K.	Earnings before taxes (EBT)	9,2	24,6%	1,0	4,6%	-50,4	-213,9%	9,1	9,0%	55,0	59,3%	38,9	30,0%	-22,8	-36,9%	-27,2	-45,0%	-28,8	-46,0%	45,2	48,2%	2,7	3,2%	-12,2	-17,1%	19,8	2,4%
L.	Net income / Net loss	6,9	18,4%	0,7	3,4%	-50,4	-213,9%	6,8	6,8%	41,2	44,5%	29,2	22,5%	-22,8	-36,9%	-27,2	-45,0%	-28,8	-46,0%	33,9	36,2%	2,1	2,4%	-12,2	-17,1%	14,9	1,8%
M.	EAT	6,9	18,4%	0,7	3,4%	-50,4	-213,9%	6,8	6,8%	41,2	44,5%	29,2	22,5%	-22,8	-36,9%	-27,2	-45,0%	-28,8	-46,0%	33,9	36,2%	2,1	2,4%	-12,2	-17,1%	14,9	1,8%

Figure B.9: P& L dam, scenario B, 1991

Scenario B	J		F		M		A		M		J	
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%
Revenues	57,8	100,0%	42,6	100,0%	28,5	100,0%	104,1	100,0%	172,1	100,0%	110,7	100,0%
A. Total operating performance	57,8	100,0%	42,6	100,0%	28,5	100,0%	104,1	100,0%	172,1	100,0%	110,7	100,0%
B. Cost of goods sold	12,9	22,3%	11,0	25,8%	10,8	38,0%	14,7	14,2%	17,4	10,1%	15,0	13,5%
C. Gross margin	44,9	77,7%	31,6	74,2%	17,7	62,0%	89,3	85,8%	154,6	89,9%	95,7	86,5%
D. Personnel expenses	27,5	47,7%	27,5	64,6%	27,5	96,5%	27,5	26,5%	27,5	16,0%	27,5	24,9%
other operating expenses	34,5	59,7%	26,6	62,3%	25,8	90,2%	35,7	34,3%	35,9	20,9%	34,6	31,3%
other operating income	6,6	11,5%	3,9	9,1%	4,1	14,2%	4,0	3,9%	4,2	2,4%	4,9	4,4%
E. EBITDA	-10,5	-18,2%	-18,6	-43,7%	-31,5	-110,5%	30,1	28,9%	95,4	55,4%	38,4	34,7%
F. Depreciation and amortisation	9,6	16,7%	9,5	22,2%	9,3	32,6%	13,1	12,6%	13,8	8,0%	13,2	11,9%
G. EBIT	-20,1	-34,8%	-28,1	-65,9%	-40,8	-143,1%	17,0	16,3%	81,5	47,4%	25,2	22,8%
H. Financial results	-5,0	-8,6%	-4,7	-11,0%	-4,7	-16,4%	-4,6	-4,4%	-4,3	-2,5%	-4,5	-4,1%
I. Income from operations	-25,1	-43,4%	-32,8	-76,9%	-45,5	-159,5%	12,4	11,9%	77,2	44,9%	20,7	18,7%
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K. Earnings before taxes (EBT)	-25,1	-43,4%	-32,8	-76,9%	-45,5	-159,5%	12,4	11,9%	77,2	44,9%	20,7	18,7%
L. Net income / Net loss	-25,1	-43,4%	-32,8	-76,9%	-45,5	-159,5%	9,3	9,0%	57,9	33,6%	15,5	14,0%
M. EAT	-25,1	-43,4%	-32,8	-76,9%	-45,5	-159,5%	9,3	9,0%	57,9	33,6%	15,5	14,0%

Scenario B	J		A		S		O		N		D		2000	
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
Revenues	122,6	100,0%	82,8	100,0%	113,5	100,0%	173,3	100,0%	56,9	100,0%	43,2	100,0%	1.107,9	100,0%
A. Total operating performance	122,6	100,0%	82,8	100,0%	113,5	100,0%	173,3	100,0%	56,9	100,0%	43,2	100,0%	1.107,9	100,0%
B. Cost of goods sold	15,5	12,6%	13,9	16,8%	15,1	13,3%	17,5	10,1%	12,8	22,6%	11,0	25,5%	167,6	15,1%
C. Gross margin	107,1	87,4%	68,9	83,2%	98,4	86,7%	155,8	89,9%	44,0	77,4%	32,2	74,5%	940,3	84,9%
D. Personnel expenses	27,5	22,5%	27,5	33,3%	27,5	24,3%	27,5	15,9%	27,5	48,4%	27,5	63,7%	330,4	29,8%
other operating expenses	31,7	25,9%	34,0	41,0%	36,2	31,9%	43,6	25,2%	35,3	62,1%	35,5	82,1%	409,3	36,9%
other operating income	5,9	4,8%	6,2	7,4%	6,3	5,6%	6,7	3,9%	7,4	13,0%	7,6	17,5%	67,7	6,1%
E. EBITDA	53,8	43,9%	13,6	16,4%	41,0	36,2%	91,4	52,7%	-11,4	-20,0%	-23,2	-53,7%	268,3	24,2%
F. Depreciation and amortisation	13,3	10,9%	12,9	15,6%	13,2	11,6%	13,8	8,0%	9,6	16,9%	9,5	21,9%	140,8	12,7%
G. EBIT	40,5	33,0%	0,7	0,8%	27,8	24,5%	77,5	44,7%	-21,0	-36,9%	-32,7	-75,6%	127,5	11,5%
H. Financial results	-4,5	-3,7%	-4,6	-5,6%	-4,5	-4,0%	-4,3	-2,5%	-6,1	-10,6%	-6,8	-15,8%	-58,7	-5,3%
I. Income from operations	36,0	29,3%	-4,0	-4,8%	23,3	20,5%	73,2	42,2%	-27,0	-47,6%	-39,5	-91,5%	68,9	6,2%
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K. Earnings before taxes (EBT)	36,0	29,3%	-4,0	-4,8%	23,3	20,5%	73,2	42,2%	-27,0	-47,6%	-39,5	-91,5%	68,9	6,2%
L. Net income / Net loss	27,0	22,0%	-4,0	-4,8%	17,5	15,4%	54,9	31,7%	-27,0	-47,6%	-39,5	-91,5%	51,6	4,7%
M. EAT	27,0	22,0%	-4,0	-4,8%	17,5	15,4%	54,9	31,7%	-27,0	-47,6%	-39,5	-91,5%	51,6	4,7%

Figure B.10: P& L dam, scenario B, 2000

Scenario C	J		F		M		A		M		J		J		A		S		O		N		D		1991	
	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
Revenues	37,5	100,0%	14,0	100,0%	15,7	100,0%	125,7	100,0%	99,3	100,0%	129,8	100,0%	54,8	100,0%	60,4	100,0%	62,6	100,0%	93,8	100,0%	77,1	100,0%	71,1	100,0%	841,8	100,0%
A. Total operating performance	37,5	100,0%	14,0	100,0%	15,7	100,0%	125,7	100,0%	99,3	100,0%	129,8	100,0%	54,8	100,0%	60,4	100,0%	62,6	100,0%	93,8	100,0%	77,1	100,0%	71,1	100,0%	841,8	100,0%
B. Cost of goods sold	10,9	29,2%	10,7	76,2%	10,7	68,2%	15,6	12,4%	14,5	14,6%	15,8	12,1%	12,8	23,3%	13,0	21,5%	13,1	20,9%	14,3	15,3%	13,6	17,7%	13,4	18,9%	158,4	18,8%
C. Gross margin	26,6	70,8%	3,3	23,8%	5,0	31,8%	110,1	87,6%	84,7	85,4%	114,1	87,9%	42,1	76,7%	47,4	78,5%	49,5	79,1%	79,4	84,7%	63,5	82,3%	57,7	81,1%	683,4	81,2%
D. Personnel expenses	27,5	73,4%	27,5	196,1%	27,5	175,3%	27,5	21,9%	27,5	27,7%	27,5	21,2%	27,5	50,2%	27,5	45,6%	27,5	44,0%	27,5	29,4%	27,5	35,7%	27,5	38,7%	330,4	39,3%
other operating expenses	31,0	82,7%	23,1	164,8%	22,3	142,1%	32,3	25,7%	32,5	32,7%	31,2	24,0%	28,3	51,6%	30,6	50,6%	32,7	52,3%	40,2	42,9%	31,9	41,3%	32,0	45,1%	368,2	43,7%
other operating income	21,7	57,7%	18,9	134,7%	19,1	121,5%	19,1	15,2%	19,2	19,4%	19,9	15,3%	20,9	38,2%	21,2	35,1%	21,4	34,2%	21,8	23,2%	22,5	29,1%	22,6	31,8%	248,2	29,5%
E. EBITDA	-10,3	-27,6%	-28,4	-202,4%	-25,8	-164,1%	69,3	55,2%	44,0	44,3%	75,2	57,9%	7,2	13,1%	10,5	17,4%	10,6	16,9%	33,5	35,7%	26,5	34,4%	20,7	29,2%	233,0	27,7%
F. Depreciation and amortisation	9,4	25,1%	9,1	65,2%	9,2	58,3%	13,3	10,6%	13,1	13,2%	13,4	10,3%	12,6	23,0%	12,6	20,9%	12,7	20,3%	13,0	13,9%	9,8	12,7%	9,8	13,7%	138,0	16,4%
G. EBIT	-19,7	-52,7%	-37,6	-267,6%	-35,0	-222,4%	56,0	44,5%	30,9	31,1%	61,8	47,6%	-5,4	-9,8%	-2,1	-3,5%	-2,1	-3,3%	20,5	21,9%	16,7	21,6%	11,0	15,4%	95,0	11,3%
H. Financial results	-7,6	-20,2%	-4,8	-34,3%	-4,7	-29,8%	-4,5	-3,6%	-6,3	-6,3%	-4,5	-3,4%	-5,4	-9,8%	-6,6	-10,9%	-8,2	-13,1%	-11,8	-12,6%	-4,6	-6,0%	-4,7	-6,6%	-73,6	-8,7%
I. Income from operations	-27,3	-72,9%	-42,4	-301,8%	-39,6	-252,3%	51,5	41,0%	24,6	24,8%	57,4	44,2%	-10,7	-19,6%	-8,7	-14,4%	-10,3	-16,5%	8,7	9,3%	12,0	15,6%	6,3	8,9%	21,4	2,5%
J. Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K. Earnings before taxes (EBT)	-27,3	-72,9%	-42,4	-301,8%	-39,6	-252,3%	51,5	41,0%	24,6	24,8%	57,4	44,2%	-10,7	-19,6%	-8,7	-14,4%	-10,3	-16,5%	8,7	9,3%	12,0	15,6%	6,3	8,9%	21,4	2,5%
L. Net income / Net loss	-27,3	-72,9%	-42,4	-301,8%	-39,6	-252,3%	38,6	30,7%	18,5	18,6%	43,0	33,1%	-10,7	-19,6%	-8,7	-14,4%	-10,3	-16,5%	6,5	7,0%	9,0	11,7%	4,7	6,6%	16,1	1,9%
M. EAT	-27,3	-72,9%	-42,4	-301,8%	-39,6	-252,3%	38,6	30,7%	18,5	18,6%	43,0	33,1%	-10,7	-19,6%	-8,7	-14,4%	-10,3	-16,5%	6,5	7,0%	9,0	11,7%	4,7	6,6%	16,1	1,9%

Figure B.11: P& L dam, scenario C, 1991

	Scenario C	J		F		M		A		M		J		J		A		S		O		N		D		2000	
		TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	TEUR	%	,	%
	Revenues	57,8	100,0%	28,4	100,0%	19,0	100,0%	130,1	100,0%	184,4	100,0%	110,7	100,0%	109,0	100,0%	82,8	100,0%	113,5	100,0%	173,3	100,0%	50,6	100,0%	43,2	100,0%	1.102,6	100,0%
A.	Total operating performance	57,8	100,0%	28,4	100,0%	19,0	100,0%	130,1	100,0%	184,4	100,0%	110,7	100,0%	109,0	100,0%	82,8	100,0%	113,5	100,0%	173,3	100,0%	50,6	100,0%	43,2	100,0%	1.102,6	100,0%
B.	Cost of goods sold	12,9	22,3%	10,8	38,2%	10,8	56,5%	15,8	12,1%	17,9	9,7%	15,0	13,5%	14,9	13,7%	13,9	16,8%	15,1	13,3%	17,5	10,1%	12,6	24,9%	11,0	25,5%	168,1	15,2%
C.	Gross margin	44,9	77,7%	17,6	61,8%	8,3	43,5%	114,3	87,9%	166,4	90,3%	95,7	86,5%	94,1	86,3%	68,9	83,2%	98,4	86,7%	155,8	89,9%	38,0	75,1%	32,2	74,5%	934,5	84,8%
D.	Personnel expenses	27,5	47,7%	27,5	96,9%	27,5	144,7%	27,5	21,2%	27,5	14,9%	27,5	24,9%	27,5	25,3%	27,5	33,3%	27,5	24,3%	27,5	15,9%	27,5	54,5%	27,5	63,7%	330,4	30,0%
	other operating expenses	31,0	53,7%	23,1	81,5%	22,3	117,4%	32,3	24,8%	32,5	17,6%	31,2	28,2%	28,3	25,9%	30,6	36,9%	32,7	28,8%	40,2	23,2%	31,9	63,0%	32,0	74,2%	368,2	33,4%
	other operating income	6,6	11,5%	3,9	13,6%	4,1	21,3%	4,0	3,1%	4,2	2,3%	4,9	4,4%	5,9	5,4%	6,2	7,4%	6,3	5,6%	6,7	3,9%	7,4	14,7%	7,6	17,5%	67,7	6,1%
E.	EBITDA	-7,1	-12,2%	-29,2	-103,0%	-37,5	-197,2%	58,5	45,0%	110,6	60,0%	41,8	37,8%	44,1	40,5%	17,0	20,5%	44,5	39,2%	94,8	54,7%	-14,0	-27,7%	-19,8	-45,8%	303,6	27,5%
F.	Depreciation and amortisation	9,6	16,7%	9,3	32,7%	9,2	48,4%	13,4	10,3%	14,0	7,6%	13,2	11,9%	13,2	12,1%	12,9	15,6%	13,2	11,6%	13,8	8,0%	9,5	18,9%	9,5	21,9%	140,8	12,8%
G.	EBIT	-16,7	-28,9%	-38,5	-135,7%	-46,7	-245,6%	45,1	34,7%	96,6	52,4%	28,7	25,9%	31,0	28,4%	4,1	4,9%	31,3	27,5%	80,9	46,7%	-23,6	-46,6%	-29,3	-67,7%	162,8	14,8%
H.	Financial results	-5,0	-8,6%	-4,7	-16,5%	-4,7	-24,6%	-4,5	-3,4%	-4,3	-2,3%	-4,5	-4,1%	-4,5	-4,2%	-4,6	-5,6%	-4,5	-4,0%	-4,3	-2,5%	-5,9	-11,7%	-6,8	-15,8%	-58,4	-5,3%
I.	Income from operations	-21,7	-37,5%	-43,2	-152,2%	-51,4	-270,2%	40,6	31,2%	92,3	50,1%	24,1	21,8%	26,4	24,2%	-0,5	-0,6%	26,7	23,5%	76,6	44,2%	-29,5	-58,3%	-36,1	-83,6%	104,4	9,5%
J.	Extraordinary results	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%	0,0	0,0%
K.	Earnings before taxes (EBT)	-21,7	-37,5%	-43,2	-152,2%	-51,4	-270,2%	40,6	31,2%	92,3	50,1%	24,1	21,8%	26,4	24,2%	-0,5	-0,6%	26,7	23,5%	76,6	44,2%	-29,5	-58,3%	-36,1	-83,6%	104,4	9,5%
L.	Net income / Net loss	-21,7	-37,5%	-43,2	-152,2%	-51,4	-270,2%	30,5	23,4%	69,2	37,6%	18,1	16,3%	19,8	18,2%	-0,5	-0,6%	20,0	17,7%	57,5	33,2%	-29,5	-58,3%	-36,1	-83,6%	78,3	7,1%
M.	EAT	-21,7	-37,5%	-43,2	-152,2%	-51,4	-270,2%	30,5	23,4%	69,2	37,6%	18,1	16,3%	19,8	18,2%	-0,5	-0,6%	20,0	17,7%	57,5	33,2%	-29,5	-58,3%	-36,1	-83,6%	78,3	7,1%

Figure B.12: P& L dam, scenario C, 2000

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